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Devoted to Science and the Mechanic Arts

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THE SELLERS OR FRANKLIN INSTITUTE STANDARD SCREW THREAD.

BY

COLEMAN SELLERS, 3rd.[1]

Member of The Franklin Institute.

Now that the Joint Commission from Great Britain, Canada and the United States has agreed upon a standard for screw threads for use in the three great English speaking countries, it is interesting to recall that The Franklin Institute had a major role in establishing the United States Standard Thread, from which was derived the American Standard, and which now becomes the International Standard with only slight modification.

In the early days of the nineteenth century when the industrial age was beginning to blossom there were no standards in general use. Each manufacturer set up his own system of tapers, shaft fits and screw threads. Indeed, even bolts made by two mechanics in the same shop might differ radically in shape of threads or in pitch. Gradually, prominent shops, and sometimes neighborhoods, evolved standards of their own which usually differed from those of their competitors and manufacturers in other locations. As late as 1868 the Brooklyn and Portsmouth Navy Yards used one system and the Boston and Philadelphia Navy Yards used an entirely different system. The two systems were not interchangeable, a fact which must have enraged and embarrassed many a Naval Officer of that time when it came to repairs.

The first progress towards standardization of screw threads was made in England. This is not surprising, as British industry was unquestionably ahead of that in the United States in those days.

In 1841 there appeared in the *London Mechanics Magazine* for October an account of a paper read by one Joseph Whitworth, Esq.,

[1] Production Manager, The Midvale Company, Philadelphia, Penna.; grandson of the Coleman Sellers and cousin of the William Sellers mentioned in this article.

(Note—The Franklin Institute is not responsible for the statements and opinions advanced by contributors in the JOURNAL.)

before the Institution of Civil Engineers, entitled, "On an Uniform System of Screw Threads." This paper,[2] which was later reprinted in the JOURNAL OF THE FRANKLIN INSTITUTE, was a plea for a uniform standard to "prevent the waste of bolts and nuts which is now unavoidable" and "to remove the confusion and delay" occasioned by the costly variety of "screwing apparatus" required. A table of pitches for different diameters of bolts was presented. This table was based on the averages then in use as determined by Messrs. Whitworth & Co., who made an extensive collection of "screw bolts from the principal workshops throughout England."

This same Joseph Whitworth, sixteen years later, in 1857, submitted a system of screw threads having a contour with 55° angles and rounded crests and valleys, with the pitches as proposed in the early paper. This was favorably received and adopted in Great Britain a few years later.

The birth of standardization in the United States was at the stated monthly meeting of The Franklin Institute held April 21st, 1864. On that occasion, William Sellers, then President of the Institute, read a paper which he called, "A System of Screw Threads and Nuts."[3] At the start of his paper he admitted that it would be desirable to adopt the same system as was adopted in England "unless grave objections can be urged against it and a better one substituted." Thereupon he proceeded to describe his system and show why it was more desirable than the Whitworth one. Without going into the details of the system proposed, suffice it to say that the principal differences lay in the form of threads which had angles of 60° instead of 55° and flat tops and bottoms instead of rounded. Furthermore, the pitches were different for certain diameters, although the majority of the smaller ones were the same.

The paper was carefully prepared and was quite in keeping with the high standard of The Franklin Institute papers. His pitches were derived by formula, his thread form was simple, practical, and easy to produce.

After the reading of the paper the following important resolution was adopted on motion by Mr. Algernon Roberts, seconded by Mr. C. T. Parry:[4]

> Resolved, that the following gentlemen be a Committee to investigate the question of a proper system of screw threads, bolt heads, and nuts to be recommended by this Institute, for the general adoption by American engineers:
>
> | | |
> |---|---|
> | William B. Bement | Bement & Dougherty (later Bement Miles & Co.) |
> | C. T. Parry | Supt., Baldwin Locomotive Works |
> | J. Vaughan Merrick | Merrick & Sons |
> | John H. Towne | I. P. Morris, Towne & Co. |
> | Coleman Sellers | Eng., Wm. Sellers & Co. |

[2] JOURNAL OF THE FRANKLIN INSTITUTE, Vol. 33, p. 327 (1842).
[3] *Ibid.*, Vol. 77, p. 344 (1864).
[4] *Ibid.*, Vol. 77, p. 351 (1864).

B. H. Bartol.....Supt., Southwark Foundry
Edward Longstreth.....Foreman, Baldwin Locomotive Works
James Moore.....Matthews & Moore
Algernon Roberts.....Pencoyd Iron Works

It is interesting to note that all of these men were prominent in the industries of their time and most of the names are closely linked with the history of The Franklin Institute.

The Committee met a number of times during the succeeding months and finally presented its report at the stated monthly meeting of the Institute held on December 15th, 1864.[5] The report was short and to the point. It told how their investigation disclosed that the varieties of threads in use were "much greater than they had supposed possible." While not mentioning the system of threads proposed by William Sellers in April of that year, the system described was the same. The report ended with three resolutions: one, that the Institute recommend the system for general adoption by American engineers; the other two, that copies of the resolutions be sent to the Chief of Bureau of Steam Engineering of Navy, Chiefs of Ordnance for the Army and Navy, Chiefs of the Engineer and Military R. R. Corps, Superintendents and Master Mechanics of all Railroad Companies, all Mechanical and Engineering Associations and Institutes, and the principal machine and engine shops in the country, with the request that they use their influence to adopt the system as standard and urging that they require all builders under any new contract to conform to the proportions recommended.

Opinions were invited, but apparently only one worthy of publishing was received. This was from Robert Briggs who, in a long and verbose paper published in the JOURNAL for February, 1865,[6] criticized both the Whitworth and the Institute systems and proposed one of his own.

The influence of The Franklin Institute was widespread and it was not many years before the new standard was recognized and accepted in many places. For instance, the Secretary of the Navy in March, 1868, created a board to recommend a standard gauge for bolts and nuts. The members were instructed to visit the principal toolmakers and machinery builders in Boston, Springfield, Providence, New York City, Newark, Philadelphia and Pittsburgh, to obtain information on the present practice. Two and a half months later, on May 15th, the board submitted a comprehensive report complete with charts, tables and drawings, recommending "the system of Mr. Sellers as the standard for the Navy." Evidently the report was satisfactory, for on the very next day the Honorable Gideon Welles, Secretary of the Navy, authorized the adoption for the Naval Service. Thenceforth, the Sellers or Franklin Institute standard became the standard for the U. S. Government, not only for the Navy but all other departments, except for small arms where special fine threads were required.

[5] *Ibid.*, Vol. 79, p. 53 (1865).
[6] *Ibid.*, Vol. 79, p. 111 (1865).

With this impetus, acceptance was speeded and the next year the Pennsylvania Railroad ordered a set of gauges and adopted the system on all of its lines. Other railroads followed and in 1870 the Master Mechanics Association, and 1872 the Master Car Builders, officially took similar action.

There are always those who object to the actions of the majority and so there were dissenters to the U. S. Standard for Screw Threads and Bolts. One of these, Mr. John L. Gill, Jr., of Philadelphia, read a paper on screw threads at the stated meeting of the Institute on November 16th, 1887.[7] He proposed a new system which, by making the threads less deep and of special shape similar to the buttress threads now used on certain applications, provided greater strength than the other systems for any given size of job. This system had its advantages and disadvantages but never came into general use. It seems sensible, if increased strength is required, to use a larger sized bolt of standard form rather than go to a special shape.

It is sometimes hard for us to realize how remote Europe of the 1880's was from the United States. That such was the case is evidenced by a letter received by the Institute in 1886 from Der Verein Deutscher Ingenieure or Society of German Engineers, of Berlin. They asked if any standard for screw threads had been adopted in the United States and if so, what standard, and to what extent it had received acceptance. They went on to refer to an article which appeared in *Engineering* (London) September, 1886, which asserted the system of threads proposed by Sellers "had not been found to fully meet the requirements of practice and had been abandoned in the United States."

Mr. William H. Wahl, then Secretary of the Institute, did not answer directly. Instead, he used a method of reply which must have been quite convincing.[8] What he did was write to the principal railroad companies and manufacturers of bolts and machinists' tools and then sent their replies to the Germans with a short note stating that "it must be evident that the Sellers or Franklin Institute Standard for pitches and form of thread is accepted and used throughout the United States to the exclusion of any other." The letters were from such companies as Pratt & Whitney Co., Pennsylvania R. R. Co., C. B. & Q. R. R., D. L. & W. R. R., Louisville & Nashville R. R., New York Central & Hudson River R. R. It is interesting to note in passing that the last-mentioned railroad operated and controlled 993 miles of railroad while the Pennsylvania R. R. had 7,346 miles, according to Mr. Wahl.

The German engineers were very grateful for the information and wrote that they were proceeding in an effort to establish a metric standard thread based on The Franklin Institute form of cross section. They also declared they could "now reply to the false representations of the

[7] *Ibid.*, Vol. 125, p. 185 (1888).
[8] *Ibid.*, Vol. 123, p. 261 (1887).

English journals, which incline rather to gratification of the national vanity than what is right and proper in the premises."

This sentence is quoted without meaning any reflection on the English publications. It is interesting and somewhat amusing to note the feeling of professional jealousy on both sides.

From the foregoing it may be seen that the standard was well established by the time of this correspondence and no serious challenge was made thereafter.

The U. S. Standard was amplified and adopted by the technical societies such as the American Society of Mechanical Engineers and known as the National Standard. Finding that finer pitches were required for automotive work, the Society of Automotive Engineers adopted their own standard with the same tooth form but with a greater number of threads per inch for given diameters. The two systems were later approved by the American Standards Association and combined in one standard which has two series, one with coarse threads (NC) and one with fine threads (NF). This is known as the American Standard.

Westinghouse Research Brings Warmer Color to Mercury Lighting.—A new method of transforming bluish-white mercury light into a warmer color without sacrificing high efficiency has been disclosed by Luke T. Thorington, engineer of the Westinghouse Lamp Research Laboratories at Bloomfield, New Jersey.

The new method involves the use of a special high-temperature phosphor to powder the inner glass wall of the mercury vapor lamp. Through fluorescence, he said, "this coating transforms invisible ultraviolet into pure red light. The blend of mercury light and phosphor light is eight times richer in red than the light from regular mercury lamps of clear glass." Then too although mercury lamps produce the most light per watt of any practical, high-intensity light source, the red deficiency of mercury light has restricted their use as sole light sources in some cases.

When the first mercury lamps were introduced, they produced a cold, blue light which distorted color values. Gradually the color of mercury light has been improved by boosting the pressure of the mercury arc inside the lamp. To do a complete color-correcting job, however, it has been necessary to mix mercury lamps and incandescent lamps in mercury lighting systems, or to add metals such as cadmium and zinc to the lamp itself. Both devices cut overall lighting efficiencies.

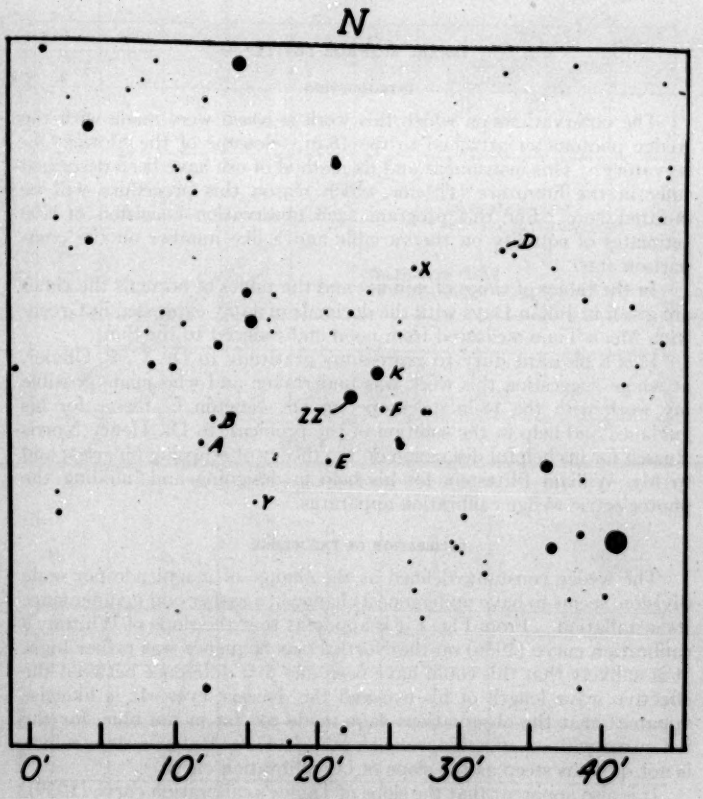
To design a fluorescent mercury lamp which retained peak efficiencies, Mr. Thorington sought a phosphor which would fluoresce under temperatures of 750°F. "No suitable phosphor ever has stood up successfully in such heat," he said, "since fluorescent lamp phosphors, for example, function best at lamp operating temperatures of only 94°F."

At first, a special fluorescent-type glass for the lamp's outer envelope was considered. In this way he hoped to capture the waste ultraviolet and convert it into useful color-correcting light, but tests showed that known fluorescent glasses fell far below the phosphor crystals in their fluorescent ability.

He then mixed and tested more than one hundred different phosphor batches, devising new methods of firing the crystals to form new phosphor combinations. When a phosphor batch was mixed that corrected color deficiencies, it failed to withstand the heat. When a batch fluoresced sufficiently, it missed the color goal. One day the properly-balanced batch was discovered, and the fluorescent mercury lamp evolved.

In the experimental models made thus far Mr. Thorington has found that a lamp with a bulging midsection, similar to an oil lamp globe, produces maximum light output and color correction because the heat level of the bulb surface coincides with the top efficiency temperature of the phosphor. In conventional tubular mercury lamp shapes, the lamp is slightly less efficient because a smaller bulb surface area causes it to operate at a higher temperature.

Mr. Thorington predicted that the lamp eventually will expand the use of high-intensity mercury lighting since in most places warmer colors of light are desirable. It will probably be used for lighting television and movie studios, where the lamp would do a flood lighting job, and for general lighting of certain factories, sports areas and other places.



ZZ Cassiopeiae.

PHOTOMETRIC RESEARCH ON THE ECLIPSING VARIABLE ZZ CASSIOPEIAE.*

BY

ISRAEL MONROE LEVITT.[1]

INTRODUCTION.

The observations on which this work is based were made with the wedge photometer attached to the 18-in. telescope of the Flower Observatory. This instrument and its method of use have been described fully in the literature (1),[2] for which reason this procedure will be omitted here. For this program each observation consisted of five estimates of equality on the variable and a like number on the comparison star.

In the tables of times of minima and the tables of normals the times are given in Julian Days with the decimals of a day expressed in Greenwich Mean Time measured from noon and reduced to the Sun.

It is a pleasant duty to express my gratitude to Dr. C. P. Olivier, at whose suggestion this work was undertaken and who made possible my work with the 18-in. telescope; to Dr. Newton L. Pierce for his guidance and help in the solution of the problem; to Dr. Henry Norris Russell for his helpful discussion on the theory of eclipsing binaries; and to Mr. William Blitzstein for his help in designing and building the photoelectric wedge calibration apparatus.

CALIBRATION OF THE WEDGE.

The wedge constant, defined as the change of magnitude per scale division, seems to have undergone a change in a rather odd manner since its installation. From Fig. 1 it is apparent that the slope of Whitney's calibration curve (1936) on the North Polar Sequence was rather high. It is unlikely that this could have been due to a difference between the effective wave length of his eye and the average eye. It is likewise apparent that the observations were made too far in the blue, for the S-8 curve with an effective wave length of $\lambda 4730$ (2), has a slope which is not quite as steep as the slope of his calibration curve.

It is also apparent that the slope of Taylor's calibration curve (1939), also made on the North Polar Sequence, was too low. In fact, it was

* A Dissertation in Astronomy presented to the Faculty of the Graduate School in partial fulfillment of the requirements for the degree of Doctor of Philosophy, at the University of Pennsylvania.

[1] Director, Fels Planetarium, The Franklin Institute; and Astronomer, Cook Observatory, The University of Pennsylvania, Philadelphia, Pennsylvania.

[2] The boldface numbers in parentheses refer to the references appended to this paper.

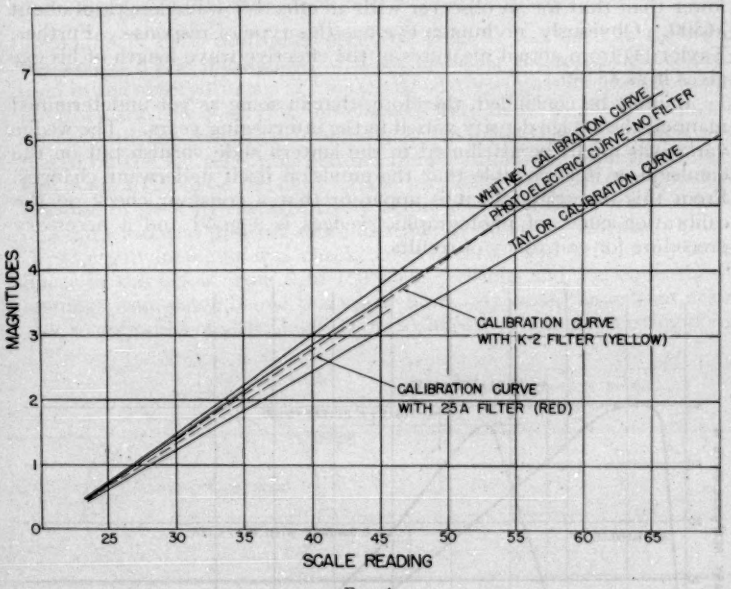


FIG. 1.

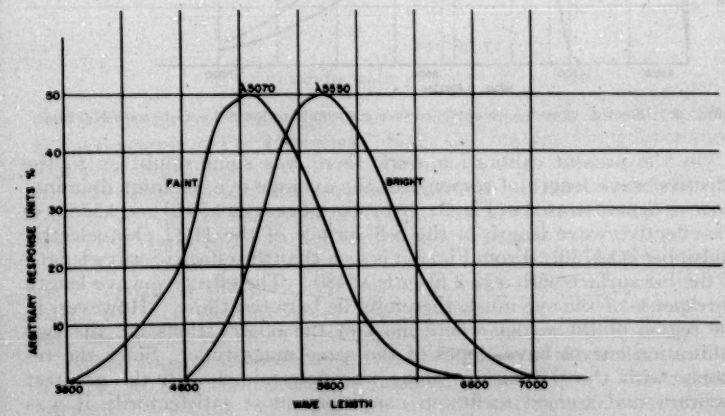


FIG. 2. Visual response curves for faint and bright illumination.

lower than that for an observer with an effective wave length of about λ6300.' Obviously, no human eye has this type of response. Further, Taylor (3) from actual measures of the effective wave length of his eye gives it as λ5180.

It must be concluded, therefore, that in some as yet undetermined manner, the wedge density varied in the intervening years. The wedge variations might be attributed to the lantern slide varnish put on the emulsion or it is possible that the emulsion itself underwent changes. From this investigation, it is apparent that a constant check on the calibration curve of photographic wedges is a good and a necessary procedure for constancy of results.

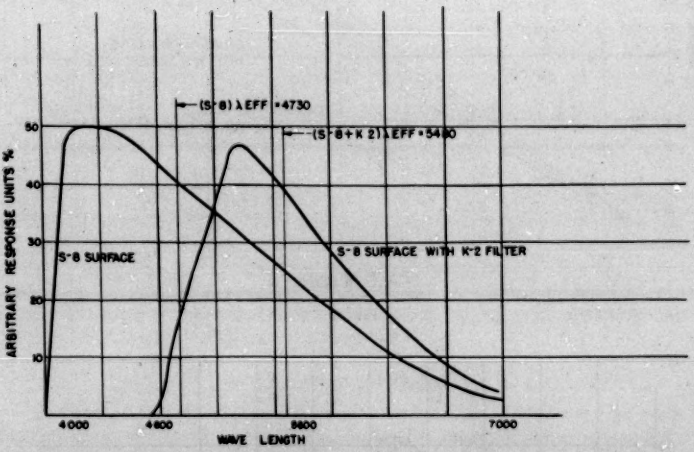


FIG. 3. Effective wave length of S-8 surface and combination S-8 surface with K-2 filter.

In the present calibration work there was some doubt as to the effective wave length of response of the average eye for faint illuminations as demonstrated in Fig. 2. At ranges between λ5070 and λ5550 (4) the effective wave length of the S-8 surface of the 1P22 photoelectric multiplier is λ4730. From Fig. 3 it is seen that the effective wave length of the S-8 surface with a K-2 filter is λ5480. The effective wave length of response of the eye must, therefore, lie between these. However, in the region of the wedge where most of the observations lie, the two calibration curves have slopes of the same magnitude. Since the response with the S-8 surface alone was determined with the greatest accuracy and seemed to fit our conditions most satisfactorily it was chosen as the final calibration of the wedge.

Procedure for Calibration.

An RCA 1P22 multiplier photocell (Fig. 4) was used to calibrate the wedge. The photometer was set up using the same light source used in the observations. To match as nearly as possible the photometer when used in observing, no changes were made in the photometer set-up with the exception of adding an aluminized mirror which was placed on the 45° reflecting plate of the photometer. This installation was made to increase the light for the very dense portions of the wedge. Because of the extreme sensitivity of the photocell the work was done in a dark room to prevent spurious readings due to light leaks.

The galvanometer was checked for linearity and was found to be linear in the region from 0 to 150 scale divisions and consequently all readings were made below this upper limit. It would have been desirable to get ratios of all scale divisions to 20 (which was the starting point

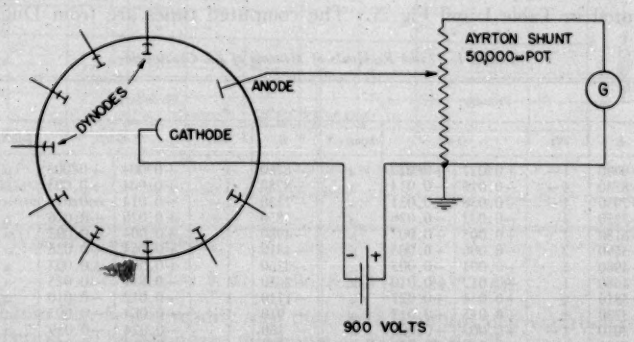


FIG. 4. Circuit for wedge calibration (RCA 1P22).

on the photometer scale) but that would have involved a difference of about 6 magnitudes or 250 times. Since the combination, galvanometer and photomultiplier, is a linear instrument, such ratios could not be directly measured with the required accuracy. It was necessary, therefore, to step the measurement through the region of the wedge in which the ratios could be measured accurately. For this reason, steps of five scale divisions on the wedge were chosen. In making the readings the dark current of the photocell was considered and a zero reading was taken for each measurement. Ten readings were taken for each step and the arithmetic mean was chosen as the most probable value.

If the apparatus necessary for the calibration of the wedge is already set up, the calibration procedure takes only a few days. This compares extremely favorably with the time usually spent on calibrating, using the stars.

From the laboratory experiments it was realized that more attention should be paid to the constancy of the light source than is usual in astronomical work. It was found that the light output varies with external temperatures. The turning on and off of an air conditioner in the laboratory almost instantly resulted in the variation in the light output. It was found that the contact resistance can change abruptly if there is any play in the lamp socket in the connection to the lamp.

The variation of *ZZ Cassiopeiae* was first disclosed by Belanowsky (5) in 1925 and its character as an eclipsing variable definitely established by Zessewitsch (6) in 1926. Subsequent observations by Martinoff (7) in 1933–1937 confirmed the period and gave estimates for the duration of the eclipse and magnitude range for primary and secondary minima.

Epoch and Period.

A summary of the available material on epoch and period is presented in Table I and Fig. 5. The computed times are from Dugan

TABLE I.—*Time Residuals of Minima of ZZ Cassiopeiae.*

Primary					Secondary				
E	Wt.	*O–C*		Source *	*E*	Wt.	*O–C*		Source *
−8990	1−	+0^{d}027	+0^{h}022	a	−8760	1	+0^{d}004	+0^{h}003	a
−8530	1−	−0.018	−0.014	a	−8280	1−	+0.004	+0.003	a
−7930	1−	−0.038	−0.031	a	−7130	1−	−0.014	−0.011	a
−7250	1	−0.032	−0.026	a	−5850	1−	+0.020	+0.016	a
−6180	1	+0.003	+0.002	a	−4980	1	+0.004	+0.003	a
−5590	2	−0.006	−0.005	a	−4310	1−	+0.044	+0.035	a
−4960	1	−0.004	−0.003	a	−3260	1	+0.004	+0.003	a
−4680	1	+0.012	+0.010	a	−2650	1	−0.031	−0.025	a
−3810	1	+0.034	+0.027	a	−1710	1	−0.012	−0.010	a
−3230	1	+0.015	+0.012	a	− 910	1−	−0.004	−0.003	a
−3010	1−	+0.003	+0.002	a	− 100	1	−0.024	−0.019	a
−2630	1	−0.008	−0.006	a	+ 700	1−	−0.012	−0.010	a
−2300	1	−0.004	−0.003	a	+1500	1−	+0.051	+0.041	a
−1560	1	−0.014	−0.011	a	+2310	2	−0.009	−0.007	a
− 900	2	−0.016	−0.013	a	+3110	3	−0.026	−0.021	a
− 70	1	−0.001	−0.001	a	+3920	2	+0.001	+0.001	a
+ 280	1−	+0.004	+0.003	a	+4380	0	−0.013	−0.010	a
+ 700	2	+0.018	+0.014	a	+5406	1	+0.004	+0.003	d
+1500	2	+0.014	+0.011	a	+5900	2	+0.009	+0.007	d
+1640	1	+0.030	+0.024	b	+7682	3	+0.009	+0.007	e
+2490	3	0	0	a					
+3110	3	−0.011	−0.009	a					
+3530	1	−0.001	−0.001	a					
+3780	2	+0.012	+0.010	a					
+3937	1	−0.013	−0.010	c					
+4280	2	+0.006	+0.005	a					
+4409	1	+0.002	+0.002	c					
+4827	1	−0.003	−0.002	c					
+5865	2	+0.013	+0.011	d					
+7682	4	+0.014	+0.011	e					

* a—Harvard College Observatory observations.
b—Zessewitsch (6).
c—Martinoff (7).
d—Taylor (9).
e—Levitt.

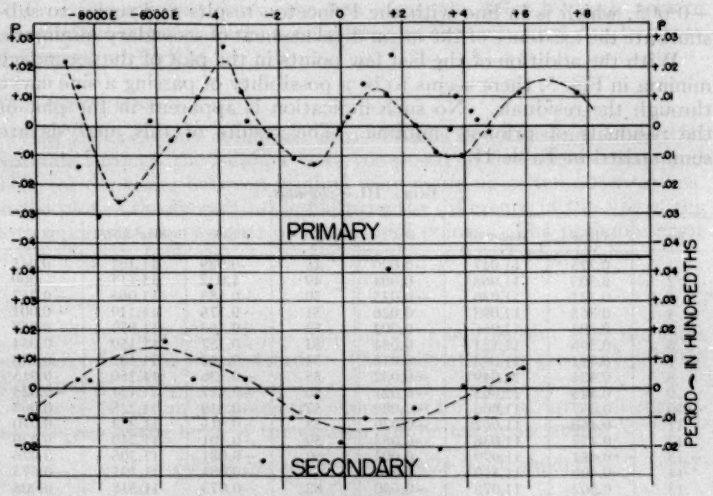


FIG. 5. Time residuals of minima.

NOTE.—The curves are not intended to show a periodicity. They are drawn in to help the eye follow the course of the variation.

and Wright's (8) elements:

$$\text{Pr. Min.} = \text{J. D. } 2422628.763 + 1.2435274E.$$

From Fig. 5, it is readily seen that there is an irregular variation in the period. The secondary minimum, as noted in the Princeton paper, appears to be almost in phase with the primary minimum, so that the variation due to a rotation of the line of apsides, appears ruled out. Further, Taylor (9) finds the residuals of secondary minimum, counted as usual from Pr. Min. $+\frac{1}{2}P$, are about -0^d010. This value is slightly higher though on the same order as that deduced by Dugan and Wright. The present work sets the residuals of secondary minimum at about

TABLE II.

	Princeton Contr. No. 19	Taylor	Present Work
Course of the Curve	Fig. 6	unchanged	unchanged Fig. 5
Primary and Secondary Variation	alike	alike	alike
Displacement of Secondary	-0^d0006	-0^d010	-0^d0005
Duration of Eclipse	0^d32	0^d31	0^d282
Normal Photo Mag.	10.84	10.83	
Visual Mag.	11.03		11.078*

* Average maximum from theoretical light curve.

$-0^d.005$, which is in line with the Princeton results and seems to substantiate the existence of the mean displacement of secondary minimum.

With the addition of the last few points in the plot of the secondary minima in Fig. 5, there seems to be a possibility of passing a sine curve through the residuals. No such indication is apparent in the plot of the residuals of primary minima. The results of this analysis are summarized in Table II.

TABLE III.—*Normals.*

No.	Phase	Magnitude	*O–C*	No.	Phase	Magnitude	*O–C*
1	0.275	11.047	−0.020	48	0.989	11.161	0.045
2	0.303	11.068	0.004	49	1.032	11.114	000
3	0.341	11.036	−0.025	50	−0.185	11.065	−0.057
4	0.365	11.093	0.026	51	−0.176	11.119	−0.001
5	0.383	11.066	−0.002	52	−0.163	11.072	−0.053
6	0.396	11.124	0.054	53	−0.157	11.159	0.034
7	0.412	11.093	0.018	54	−0.142	11.102	−0.028
8	0.434	11.049	−0.032	55	−0.136	11.150	0.015
9	0.445	11.062	−0.021	56	−0.127	11.124	−0.023
10	0.457	11.066	−0.022	57	−0.119	11.225	0.068
11	0.466	11.062	−0.029	58	−0.111	11.253	0.070
12	0.472	11.038	−0.053	59	−0.101	11.240	0.029
13	0.481	11.099	0.002	60	−0.081	11.306	0.076
14	0.490	11.110	0.010	61	−0.084	11.191	−0.073
15	0.502	11.078	−0.030	62	−0.073	11.318	0.008
16	0.512	11.157	0.037	63	−0.063	11.206	−0.134
17	0.521	11.153	0.021	64	−0.056	11.324	−0.048
18	0.534	11.205	0.055	65	−0.049	11.377	−0.023
19	0.551	11.129	−0.042	66	−0.043	11.437	0.002
20	0.564	11.232	0.041	67	−0.039	11.400	−0.041
21	0.578	11.227	0.014	68	−0.030	11.498	0.026
22	0.591	11.216	−0.010	69	−0.025	11.467	−0.016
23	0.600	11.256	0.021	70	−0.019	11.461	−0.031
24	0.612	11.228	−0.014	71	−0.014	11.563	0.060
25	0.620	11.272	0.026	72	−0.005	11.604	0.094
26	0.632	11.267	0.023	73	−0.002	11.509	−0.003
27	0.642	11.222	−0.018	74	0.011	11.599	0.093
28	0.654	11.250	0.018	75	0.026	11.388	−0.087
29	0.663	11.259	0.036	76	0.049	11.361	−0.086
30	0.681	11.218	0.017	77	0.047	11.347	−0.058
31	0.691	11.103	−0.085	78	0.056	11.406	0.039
32	0.700	11.158	−0.017	79	0.065	11.334	0.007
33	0.710	11.172	0.010	80	0.074	11.352	0.062
34	0.722	11.126	−0.022	81	0.080	11.317	0.050
35	0.731	11.116	−0.021	82	0.087	11.365	0.135
36	0.740	11.199	0.067	83	0.093	11.215	000
37	0.749	11.115	−0.009	84	0.098	11.271	0.076
38	0.759	11.086	−0.033	85	0.104	11.155	−0.021
39	0.770	11.148	0.031	86	0.109	11.071	−0.089
40	0.785	11.136	0.021	87	0.114	11.218	0.070
41	0.801	11.149	0.039	88	0.120	11.197	0.062
42	0.819	11.089	−0.017	89	0.129	11.133	0.021
43	0.858	11.123	0.021	90	0.140	11.075	−0.031
44	0.905	11.077	−0.021	91	0.153	11.124	0.024
45	0.929	11.080	−0.019	92	0.172	11.107	0.013
46	0.945	11.065	−0.034	93	0.201	11.104	0.020
47	0.962	11.115	0.014	94	0.228	11.098	0.020
				95	0.248	11.047	−0.024

NOTE.—Column 1 is the normal number, column 2 is the phase in days, column 3 is the magnitude, and column 4 is the residual from the theoretical light curve (*O–C*).

From the present work, the new elements differ by such minute quantities that for prediction purposes the elements used can be those of Dugan and Wright.

The Observations.

The observations combined into normals are listed in Table III. Normals through the minima were composed of five observations and those for the times between minima were made up of ten observations. In the plot of the theoretical light curve the difference in the size of the dots represents the weights of the normal points. The probable error of the normals from the theoretical curve is ±0.030 magnitudes.

Discussion.

From an inspection of the light curve it is apparent that the light between minima is not constant. The light at 90° is greater by $0^m.036$ than that at 270°.

For rectification purposes the expression for the variation disregarding eclipses of the light of the system can be taken as (10):

$$I = I_0(1 + A_1 \cos\theta + A_2 \cos 2\theta + B_1 \sin\theta), \qquad (1)$$

where the sine term is arbitrarily inserted as an indication of any asymmetry. I_0 in practice is never known accurately until the solution of the least squares problem. Therefore this equation is unsuitable for use. However Eq. 1 may be written in the form:

$$I = I_R + \Delta I,$$

where

$$\Delta I = A_0' + A_1' \cos\theta + A_2' \cos 2\theta + B_1' \sin\theta,$$

and I_R equals 1 corresponding to some convenient reference magnitude.

It is seen that:

$$I_0 = I_R + A_0',$$

$$A_1 = \frac{A_1'}{I_0}, \qquad A_2 = \frac{A_2'}{I_0}, \qquad B_1 = \frac{B_1'}{I_0}.$$

From a least squares solution of observation equations of the above form, the values of the coefficients A_0', A_1', A_2', and B_1' and their probable errors are found to be:

$$A_0' = -0.034 \pm 0.004,$$
$$A_1' = -0.007 \pm 0.005,$$
$$A_2' = -0.024 \pm 0.006,$$
$$B_1' = +0.016 \pm 0.003.$$

The small probable error in B_1' indicates that the sine term is real and is a true measure of the asymmetry.

The reflection coefficient was very small, being on the order of its probable error. The ellipticity coefficient A_2 is real but quite small. With this value of A_2; Z, the ellipticity, defined by (10)

$$Z = \frac{4e}{c}(-A_2 + 0.006),$$

is derived as 0.042.

In the rectification procedures both the reflection effect and the ellipticity effect were added, despite the fact that rectification for ellipticity is considered a division procedure. Because A_2 is so small, the difference between the rectified value obtained by division and that by addition is less than 1 per cent of the total light in the worst possible case and is generally less. For this reason a rectification by addition alone was performed.

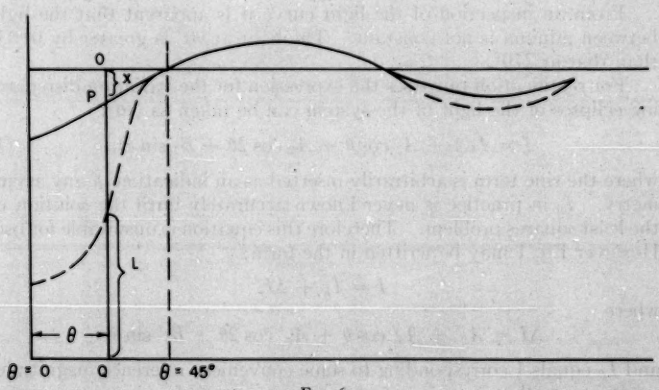


FIG. 6.

The following analysis using Fig. 6 will justify the procedure:

where the ordinate $OQ = 1 =$ mean light of system,
where the ordinate $OP = x = |A_2 \cos 2\theta|$, the ellipticity effect.

The rectification factor by division is $\frac{1}{1-x}$,

$$L_R = L_u\left(\frac{1}{1-x}\right), \tag{2}$$

where L_u is the unrectified light. In this work the rectified light was considered as

$$L_R = L_u + x. \tag{3}$$

To find the variation in Eqs. 2 and 3, $\left(\frac{1}{1-x}\right)$ was expanded in a

power series and, if second order terms are neglected, Eq. 2 becomes

$$L_R = L_u(1 + x).$$

The greatest value of x is 0.024 and occurs at $\theta = 0$ and at primary minimum

$$L_u = 0.712.$$

Therefore, Eq. 2 results in a value for L_R of 0.729. Equation 3 results in a value for L_R of 0.736; the difference is 0.007 which is less than 1 per cent of the total light of the system.

Putting a theoretical light curve through the normals at primary minimum proved to be rather difficult. From the theoretical light curve it can be seen that the bottom of the curve at primary minimum is rather high compared to three normals which are at the low point. Fitting the curve proved difficult because when the curve was passed through these points the shoulders pulled away from the normals thus tending to shorten the eclipse. When the shoulders were catered to, the depth was such as to miss the three normals. Finally, the shoulders were pulled in as much as possible without departing radically from the normals outlining this part of the curve. Fortunately the secondary minimum was also of help in establishing the time of beginning of eclipse. In this fashion a series of compromises was used to arrive at a satisfactory theoretical curve and the subsequent solution. A check using the equality of the sum of the squares of the residuals as a criterion was made and the validity of the curve was established.

The spectrum of *ZZ Cassiopeiae* was observed by Struve [3] on seven plates. He says the star "is about B2 or B3, with very diffuse lines of helium and fairly strong and broad lines of hydrogen." From the ratio of surface brightnesses and with the primary considered a B2 star, the secondary turns out to be about a B7 star. From this the *ZZ Cassiopeiae* system may be considered as consisting of two normal main sequence objects. The mass ratio is computed from the relation (11)

$$\frac{m_b}{m_f} = \left(\frac{L_b}{L_f}\right)^{2/5},$$

which holds for main sequence stars.

After the solution was completed it was checked with the coefficients of reflection and ellipticity as defined by Russell (10) in his paper on intermediary elements.

It was discovered that A_1 given by

$$A_1 = 0.347\left(\frac{E_2}{E_1}L_1r_2^2 - \frac{E_1}{E_2}L_2r_1^2\right)\sin i$$

turns out to be -0.070 compared with -0.007 ± 0.005 obtained from a least squares solution.

[3] Private communication.

Likewise, A_2 defined as

$$A_2 = 0.073\left[\left(\frac{E_2}{E_1}L_1r_2^2 + \frac{E_1}{E_2}L_2r_1^2\right) - \tfrac{1}{2}(C_1L_1 + C_2L_2)\right]\sin^2 i$$

is -0.046 compared with -0.024 ± 0.006 determined from the least squares solution.

Here we find a discrepancy in the theoretical values on the order of several hundred per cent in the reflection coefficient and about one hundred per cent in the ellipticity coefficient.

The problem is now to alter the physical elements of the system, without destroying the light curve which defines the observations, to bring the observed and theoretical values of A_1 and A_2 in concordance.

At the suggestion of Dr. John B. Irwin, the value of A_2, the ellipticity coefficient, was increased by three times its probable error and this was used as the basis of a new solution. The rectification was performed as indicated previously and a new curve was put through the normals.

The depths at both primary minimum and secondary minimum were altered slightly. $1 - \lambda p$ which was 0.288 became 0.274 and $1 - \lambda s$ which was 0.102 became 0.090.

The elements changed considerably as indicated in the summary of results.

However, when A_1 and A_2 were again computed, the value of A_1 was once more seen to be out of line. The value of A_2 which was used in the solution as -0.042 resulted in a theoretical value of A_2 of -0.048. This may be considered in very close agreement. The value of A_1 used in the solution was fixed at -0.007 and the derived theoretical value was -0.072. This was slightly greater than the previously derived value of A_1.

The reason for this anomalous behavior of A_1 is apparent from a study of the equation defining that coefficient. To reduce A_1 it was desirable to increase the radii through k. This was done, but unfortunately as k became greater the differences in the lights of the two stars increased so that any advantage gained through an increase in the radii was overcome by the L factor.

When the luminous efficiencies were computed for the stars, it was seen that these were the principal factors influencing the final results. Attempts were made to get favorable E's by choosing the spectral class which would give the smallest ratio of the luminous efficiencies. As *ZZ Cassiopeiae* was considered either a B2 or B3 star its companion was derived as a B7 or a B8 star. Some advantage was gained here for in the solution using $A_2 = -0.042$, the derived value of A_1 for a B2–B7 combination was -0.072, while for a B3–B8 combination A_1 was computed as -0.080. If it were possible to eliminate E_1 and E_2

or if the spectral class of the primary was about an FO, then the ratio $\frac{E_1}{E_2} = \frac{E_2}{E_1} = 1$. In this case the derived values of A_1 and A_2 are $A_1 = -0.021$, $A_2 = -0.054$. This might be considered satisfactory agreement. It should be noted that Russell (10) employs this stratagem in arriving at the equation for ellipticity.

However, we are dealing with early B stars and the reflection coefficient derived from a least squares solution is admittedly small.

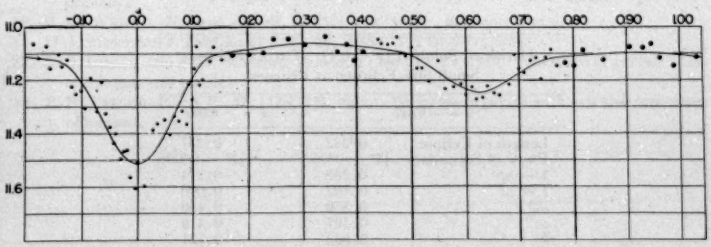


FIG. 7.

Detailed discussions were held with Dr. Henry Norris Russell in attempts to find reasons for the discrepancy of these coefficients. Dr. Russell said, "The visual light curve is certainly an excellent one. In this case where observational evidence disputes the theory it is best to adhere strictly to the observational evidence and completely forget the theory." Dr. Russell admits his theory is rudimentary and based on approximations and for this reason too close an agreement cannot be expected.

The derived value of A_1 presents a challenge to the theory and the answer will come at some future date when enough observations have been made either to confirm or dispute this value or when the theory has been validated by a wealth of supporting material.

This paper marks the end of an era for the Flower Observatory in that *ZZ Cassiopeiae* probably will be the last eclipsing variable observed visually. As these results are being compiled the test program of a Pulse Counting Photoelectric Photometer gets under way and the writer sincerely hopes to publish at some early date a summary comparing the elements of *ZZ Cassiopeiae* derived from visual observations and those derived from photoelectric observations.

SUMMARY OF RESULTS.

Designation	see note
R. A. 1900	$0^h\ 27^m\ 50^s$
Dec. 1900	61° 57′ .8
Spectral type primary	B2
Spectral type secondary	B8
Maximum [a]	11.082
Primary minimum	11.248
Comparison star	star "A" on photograph
Epoch	242 2628.763
Period	$1^d.2435274$

Darkened Solution ($x = 0.4$). Partial Eclipse.
Small Star Eclipsed at Primary.

	$Z = -0.042$	$Z = -0.090$
Length of Eclipse	$0^d.282$	$0^d.287$
Phase of Secondary	$\frac{1}{2}P - 0^d.005$	$\frac{1}{2}P - 0^d.005$
$1 - \lambda_p$	0.288	0.274
$1 - \lambda_s$	0.102	0.090
α_0'	0.550	0.450
α_0''	0.507	0.416
k	0.633	0.721
L_1 [b]	0.476	0.391
L_2 [b]	0.524	0.609
J_b/J_f [b]	2.748	2.997
i	67°.3	66°.2
r_b	0.281	0.314
r_f	0.444	0.435
ρ_b [c]	0.201	ρ_b [d] 0.153
ρ_f [c]	0.049	ρ_f [d] 0.048

NOTE.—Preliminary designation is 5.1925. Alternate is S.V.S. (Soviet Variable Star) No. 54. Star "K" on photograph is BD + 61°113.

[a] Average maximum.
[b] Rectified.
[c] Density, Assumption $m_b = 1.04m_f$.
[d] Density, Assumption $m_b = 1.159m_f$.

REFERENCES.

(1) CAROLINE E. FURNESS, "Introduction to Study of Variable Stars," Houghton Mifflin (1915).
(2) RCA-1P22 Multiplier Phototube Manual.
(3) P. H. TAYLOR AND R. S. ALEXANDER, Publications of the University of Pennsylvania, *Astronomical Series*, Vol. VI, Pt. I, II, and III (1940).
(4) R. A. HOUSTOWN, "Vision and Color Vision," Longmans, Green (1932).
(5) J. BELANOWSKY, *A.N.*, Vol. 224, p. 408 (5375) (1925).
(6) W. ZESSEWITSCH, *B.Z.*, Vol. 18 (1926); *Bull. Mirov.*, No. 1 (14) (1926).
(7) D. J. MARTINOFF, *Engelhardt Publ.*, No. 20 (1938).
(8) R. S. DUGAN AND F. W. WRIGHT, *Princeton Contributions*, No. 19 (1939).
(9) P. H. TAYLOR, *P.A.S.P.*, Vol. 53, p. 117 (1941).
(10) H. N. RUSSELL, *Ap. J.*, Vol. 102, p. 6 (1945).
(11) H. N. RUSSELL, R. S. DUGAN, AND J. Q. STEWART, *Astronomy*, Vol. II, p. 717 (1938).

BIBLIOGRAPHY.

1. J. BELANOWSKY, *A.N.*, **224**, 408 (5375), 1925. Discovery of eclipsing type.
2. W. ZESSEWITSCH, *B.Z.*, **18**, 1926. Confirmed.
3. W. ZESSEWITSCH, *Bull. Mirov.*, 1 (14), 1926. Duration of primary.
4. P. P. PARENAGO, *N.N.V.S.*, **3**, 1–2 (25–6), 1930. 2 observations.
5. W. ZESSEWITSCH, *A.A.c.*, **1**, 116, 1930. Elements. List of 7 minima with *O–C*.
6. *N.N.V.S.*, Bd. **3**, No. 1–2 (25–6), p. 22, 1930. Abstract of Zessewitsch.
7. D. B. MCLAUGHLIN, *A.J.*, **38**, 45 (892), 1927. Limiting mean densities.
8. R. S. DUGAN AND F. W. WRIGHT, *A.J.*, **46**, 148 (15/1071), 1937. Variation of period. Elements for prediction.
9. D. J. MARTINOFF, *N.N.V.S.*, **5**, 205 (7/55), 1938. Range in magnitude. Duration of primary.
10. F. W. WRIGHT, *Harvard Annals*, **89** (10), 171, 1937. Sequence and chart.
11. D. J. MARTINOFF, *Engelhardt Bull.*, **20**, 1938. 571 Observations.
12. R. S. DUGAN AND F. W. WRIGHT, *Princeton Cont.*, **19**, 24, 1939. Variation of period. List of minima and graph.
13. P. H. TAYLOR, *P.A.S.P.*, **53**, 117 (312), 1941. Variation in period with new observed times of minima.

Report on Fluorine Corrosion.—A four-page technical report on fluorine corrosion is being offered in reprint form by The International Nickel Company, Inc., 67 Wall Street, New York 5, N. Y. Written by W. R. Myers and W. B. DeLong of E. I. duPont de Nemours & Company, Inc., it discusses high temperature attack on metals by fluorine and by hydrogen fluoride and on the resistance of nickel, Monel, and other materials to these corrosive agents at various temperatures. It is illustrated by charts and includes a table of rates of corrosive attack on the materials discussed.

Cleansing and Finishing Machine Uses Wet Abrasive Principle (*Materials and Methods*, Vol. 28, No. 6).—A new machine for wet abrasive blast cleaning and finishing of metal surfaces is now available from Armstrong Chemical and Machine Co., Painesville, Ohio. The machine removes rust, scale and undesirable metal particles, such as feather edges on sharpened tools, by projecting a slurry of fine abrasive suspended in water against the surface to be cleaned. The permissible variation in abrasive size (60 to 1250 mesh) makes it commercially possible to produce finishes as low as 2–3 microinches r.m.s.

The method is said to produce a matte finish with practically no removal of metal. Specifications of the standard models are: a 30- by 30-in. cabinet, 77 in. high, one blast nozzle and exhaust blower directly connected to a ¼-hp. motor; and a 48- by 42-in. cabinet, 108 in. high, one to eight blast nozzles and exhaust blower directly connected to a 1-hp. motor. Cabinet and frame are welded steel construction.

The siphon jet principle is used for transferring the grit slurry, thus eliminating any pumps or moving parts coming into contact with the abrasive liquid. The only wearing parts are the blast nozzles. Typical applications of the cleaning machine are removal of rust, mill scale and heat treating scale; cleaning forging dies and metal molds; honing metal cutting tools; and preparation of metal parts for plating, painting or enameling.

Recording Vibrometer (*Power Generation*, December, 1948).—A new recording vibrometer which measures and records frequency, displacement, and wave shape of mechanical vibration, has been announced. Built to operate either when mounted on a fixed base or held in the hands, the vibrometer weighs only 7 lb. and is less than 8 in. in length. It was developed for application in testing all types of reciprocating and rotating machinery within a vibration frequency range of 10 to 120 cycles per second. It records both steady-state and transient vibrations. A prod extending from one side of the vibrometer is set in motion when held against a vibrating body. This motion is amplified by a cross-spring arrangement and transmitted to a stylus which inklessly records the vibration on wax paper, thus making available a permanent record for vibration analysis of equipment. Another stylus produces a timing mark near the edge of the wax paper every one-third of a second. Both the chart speed and the interval between timing marks are governed by a synchronous motor operated from a 115-v., 60-cycle power supply. Two push buttons are provided to give chart speeds of 1 in. per second and 3 in. per second. Motion of the stylus on the chart can be observed through a window in the top of the all-aluminum case. One side of the case can be removed easily, for rapid replacement of the wax paper roll when its 50-ft. capacity is depleted (General Electric Co.).

AN ENGINEERING CORRELATION OF ROOM COLORS.

BY

PARRY MOON[1] AND DOMINA EBERLE SPENCER.[2]

1. INTRODUCTION.

In the past, the assumption has generally been made that room lighting and interior decoration are separate and distinct. The colors of room surfaces, furniture, and machinery have been a matter of whim and the lighting equipment has been chosen without regard to the reflectances and hues of the room surfaces.

Today a revolution is taking place in the attitude toward color in industry. The realization is growing that *the true problem is the creation of a suitable environment.* Light and color are merely two tools for the production of this environment, and they cannot be divorced if a satisfactory effect is to be obtained.

One aspect of this new awareness of the correlation between light and color is the employment of higher reflectances for ceiling, walls, floor, and furniture. Another aspect is the increased attention paid to hue in factories and other work places. The traditional office, schoolroom, or factory was a dreary place with its uniformly drab colors and its reflectances of 0.02 to 0.10. The modern trend is toward larger windows, more artificial light, and an increased use of pastel colors.

Considerable experimental research has been done in the selection of suitable colors for industrial applications (1).[3] Recent theoretical investigations (2) are also helpful because they allow a predetermination of reflectances for good vision. In this paper we have attempted to correlate all of this recent work and to employ it in making specific recommendations for the colors to be used in rooms.

2. OBJECTIVES.

The objectives to be attained by a modern correlation of light and color are as follows:

1. Improved vision produced by *lower helios ratios* in the visual field. The traditional interior was a place of glaring contrasts between bright light-sources and dark surroundings. Helios ratios were often greater (3) than 1000:1. Best modern practice (4) keeps the adaptation helios ratios within 3:1. This promotes visual comfort and reduces eyestrain.

2. Modern methods produce *spacial lighting* similar to that obtained outdoors (5). With the older conditions, incident pharosage of a sur-

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[2] Brown University, Providence, Rhode Island.

[3] The boldface numbers in parentheses refer to the references appended to this paper.

face (6) depended to a marked extent on its position and orientation. With modern conditions, this variation is greatly reduced, thus improving visual conditions.

3. *Controlled contrast* is obtained between the details of the work and their background. Discrimination of details is possible only when light reflected from the details can be distinguished from light reflected from the background. In reading, the necessary contrast in the foveal field is obtained by the use of black print on white paper. In many industrial tasks, however, such contrast is not present. For these cases, visual conditions can often be improved by the judicious use of difference in hue (7).

4. For a given lighting system, improved reflectances give a notable *increase in usable light*. Ordinarily, a change from the conventional painting scheme to modern colors of high reflectance doubles the amount of light on the work with no increase in the cost of lighting (8). Thus the modern scheme would be desirable economically, even if there were no other advantages.

5. The use of high reflectances tends to make objects more easily visible, and the planned use of different hues aids this process. Thus there is a tendency for the new method to *increase safety* in industry. The employment of lighter colors also makes for greater cleanliness and better hygiene.

6. When the room surfaces are being refinished, there is little or no additional cost in choosing colors that are inherently attractive and in arranging them harmoniously. The result is a *beautiful color combination*.

7. High reflectances and pleasing hues produce a *bright and cheerful effect*, which tends to make people feel happier and work better.

Seven objectives have been listed. In a given case, one of these objectives may be paramount; while in another case, a different factor may be important. Generally, however, all are interrelated and the achievement of one objective will tend automatically to give the others.

3. PROCEDURE.

Now consider the establishment of methods and criteria by which the foregoing objectives can be attained. To realize a satisfactory environment, we must have first of all a sufficient amount of light, either natural or artificial. Windows must be of ample size and properly placed, and the artificial lighting system must produce sufficient light to allow good vision. More important than the amount of light, however, is the *helios distribution* in the entire field of view.

Details of fenestration and lighting are beyond the scope of this paper (6). All that need be said here is that the helios of the source, viewed from any direction, should not exceed three times the helios of the work (4). This definitely excludes all exposed lamps, both in-

candescent and fluorescent. It excludes lens plates, downlights, troffers and RLM reflectors. It excludes all louvered fixtures. It excludes the louverall ceiling. One becomes increasingly aware that for best vision there are only two satisfactory lighting systems:

(*a*) Ceiling lighting in which the entire white ceiling becomes a secondary source because of light thrown onto it (Type IIb lighting (6)).

(*b*) Ceiling lighting in which a hung ceiling of diffusing material is luminous because of lamps mounted above it (Type IIa lighting).

In many cases, the designer is not at liberty to change the windows or the lighting system but he can repaint the room. Fortunately, the selection of satisfactory colors will improve conditions, even with the

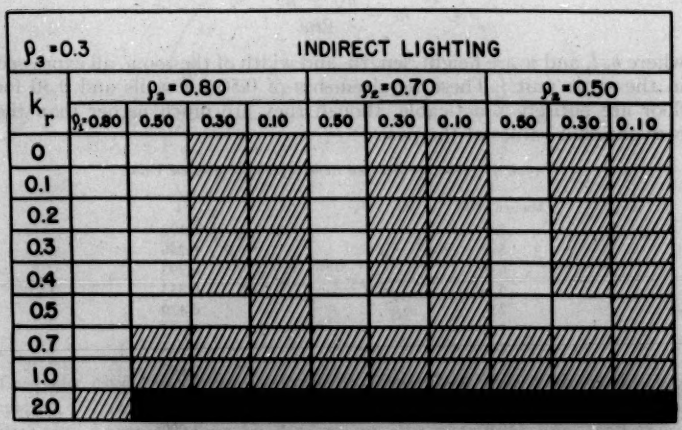

FIG. 1. Helios ratio chart, calculated for rooms with ceiling lighting (IIa or IIb). Floor reflectance = $\rho_3 = 0.30$, wall reflectance = ρ_1, ceiling reflectance = ρ_2. Clear areas indicate that the 3:1 helios ratio is satisfied, cross-hatched areas indicate that the highest ratio is between 3 and 10, black areas designate a ratio of more than 10.

poorest lighting systems. The environment will not be as satisfactory as if the best lighting were used, but bad lighting fixtures plus a modern color scheme is better than the same fixtures in drab surroundings.

The first problem is, therefore, the choice of suitable reflectances for ceiling, walls, and floor. In the past, this choice was largely arbitrary. But the recent development of the interflection method (9) allows the predetermination of reflectances that will be ideal visually. The most effective part of the room from the lighting standpoint is the ceiling, so its reflectance should be as high as possible. A reflectance of 0.80 for a matt white surface can be maintained in practice, and this value should

be considered as a minimum. In other words, ceilings should always be pure white, never cream, tan, or other tints.

An analysis of rooms by the interflection method has been summarized in a series of charts (9) which show that unless the wall reflectance is at least 0.50 and the floor reflectance is at least 0.30, the desired 3:1 helios ratio cannot ordinarily be obtained. For ceiling lighting and a floor reflectance of 0.10, the 3:1 ratio is never obtainable, even for an infinite room. If the floor reflectance is raised to 0.30, however, Fig. 1 shows that a wall reflectance of 0.50 satisfies the helios criterion except for rooms that are high compared to their width. The vertical scale in Fig. 1 refers to the shape of the room, which is specified by the *domance:*

$$k_r = \frac{h(l + w)}{2lw},$$

where h, l, and w are height, length, and width of the room, all expressed in the same unit. These requirements of 0.50 for walls and 0.30 for floor are entirely practicable, though they are much higher than the conventional values of the past.

TABLE I.—*Relation Between Reflectance and Munsell Value.*

Munsell Value	Reflectance ρ
5.0	0.193
5.5	0.240
6.0	0.293
6.5	0.353
7.0	0.420
7.5	0.495
8.0	0.576
8.5	0.667
9.0	0.767
9.5	0.878
10.0	1.000

Having raised the room reflectances, one becomes unpleasantly conscious of the customary dark furniture. Again something must be done to keep within the 3:1 helios ratio, which means the use of at least 0.30 reflectance for desks, filing cabinets, computing machines, typewriters, factory benches and machine tools. Details and trimming can still have low reflectance, in order to provide a pleasing variety, but surfaces subtending large solid angles at the eye must have high reflectances.

For example, steel office desks have been made customarily with linoleum tops having reflectances (10) of 0.05. Such surfaces offer too much contrast with white paper. The trouble is remedied by employing a linoleum with reflectance of 0.30 to 0.50. The remainder of the desk is less important visually, but the appearance of the redecorated room will generally be improved if the average desk reflectance is raised to

at least 0.30. Traditional olive-green filing cabinets have had reflectance (10) of 0.08. Typewriters and other office machines have usually been black. Most factory machinery has been black or dark gray. All of these reflectances should be raised to satisfy the 3:1 helios criterion. An alternate way of specifying reflectance is to employ *Munsell* value (11). The conversion is given in Table I.

We have considered the choice of reflectances to allow a 3:1 helios ratio. The high reflectances promote good vision and reduce eyestrain.

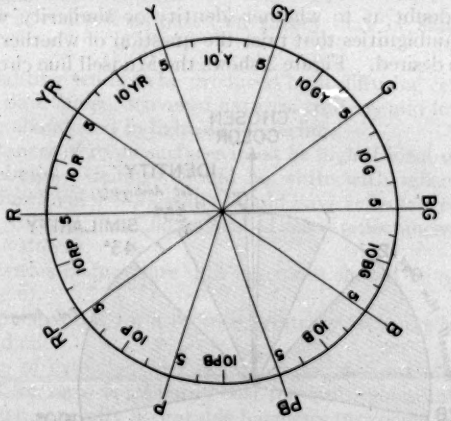

FIG. 2. Munsell hue circle, showing the ten principal hues.

They also give a maximum amount of usable light from a given lighting system. The next question deals with the selection of satisfactory hues. Beautiful harmonies can be devised on any dominant hue, and there seems to be no scientific reason for choosing one rather than another. Some decorators advocate the use of warm colors in north rooms and cool colors in south rooms. We see little value in such a procedure, however, and feel that choice of hue should be left to the users of the room.

Next consider the choice of chroma. The principle to be satisfied is that *surroundings must be unobtrusive* in rooms where close visual work is to be done. This requirement means, of course, that the helios ratio between any extended source and the work must be low (3:1). It also means that the chroma of these surfaces must be low. According to the theory of area balance (12), the product of solid angle of a color and its moment arm about the adaptation point in Munsell colorspace should be the same for each part of the visual field that is to be equally prominent. This means that high chromas are allowable on small areas

but that walls and other large areas must have low chroma to prevent them from being too prominent. Tests made on "synthetic photographs" (13) and on actual rooms (14) showed that large surfaces should not have chromas above 4. For most purposes, a chroma of 2 seems best.

Having chosen the dominant hue for a room, one may select a harmonious set of colors. As pointed out previously (15), this choice should satisfy the requirement of *similarity* or *contrast* in hue, value, and chroma. To be avoided are ambiguities: first-ambiguities that engender a doubt as to whether identity or similarity was intended, and second-ambiguities that raise the question of whether similarity or contrast was desired. Figure 2 shows the Munsell hue circle, and Fig. 3

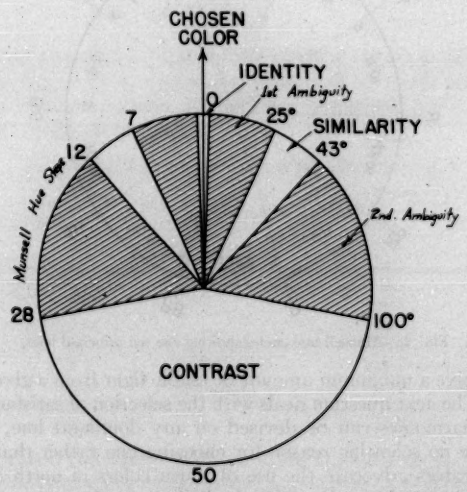

FIG. 3. Regions of similarity, contrast, and ambiguity of hue. Figure 3 is used with the hue circle of Fig. 2.

indicates the regions of ambiguity, similarity, and contrast (15). Thus, after arbitrarily choosing a hue for the walls, the designer can obtain harmonious hues for floor, furniture, and other objects by superposing Figs. 2 and 3.

One of the most satisfactory color harmonies employs two or more colors of the same hue and chroma but different values. Geometrically, this harmony is represented by several points on a vertical line in color-space (15). The only other simple harmony at constant hue, which is suitable for rooms, is an arrangement of colors on a line sloping from

high value and low chroma to low value and high chroma. A horizontal line (constant value and variable chroma) gives an undesirable effect, while a line sloping upward to the right violates the requirement of low chroma for walls. Either the vertical or the sloping line gives a beautiful effect. For rooms, however, the vertical line (constant hue and constant chroma) has the advantage of giving a less obtrusive floor. Additional interest and variety may be obtained if desired by adding an area of contrasting hue to the vertical-line harmony of constant hue, constant chroma.

The practical conclusions based on theory may be summarized as follows:

(*a*) Best lighting tends to be produced by a diffusing ceiling of uniform helios. Bare lamps, louvered fixtures, troffers, and louverall ceilings should be abandoned in lighting for seeing.

(*b*) Reflectances of room surfaces must be high if ideal visual conditions are to obtain. Ceilings should be white with reflectance of at least 0.80 (Munsell value 9.2), walls should have reflectances of at least 0.50 (Munsell value 8), and floors should have reflectances of at least 0.30 (Munsell value 6).

(*c*) Reflectances of furniture and machines should be at least 0.30 (Munsell value 6).

(*d*) The principal hue for a room is arbitrary and may be chosen to suit the individual.

(*e*) Chroma of extended surfaces must be low (not more than 4). A chroma of 2 (or zero) is probably best for walls, floor, and furniture.

(*f*) The most generally acceptable harmony for rooms is a harmony of constant chroma and constant hue (vertical line in Munsell color-space). Additional interest may be obtained by adding an area of contrasting hue.

4. SOME COLOR COMBINATIONS FOR ROOMS.

The foregoing principles have been utilized in specifying definite color combinations for rooms. The following Munsell hues (16) were used: R, YR, Y, GY, G, BG, B, PB. That is, the principal hues of the Munsell system were employed, with the exception of P and RP. The latter colors are beautiful but would not be favored by most people for industrial interiors. Yellow is employed only at high values, since the lower values of this hue are not pleasing.

Twenty-five suggested harmonies are listed in Tables II and III. Table II makes use of hues that are particularly harmonious with unstained-wood furniture. Or painted furniture of Y 8/2 or YR 8/2 may be substituted for the unstained wood. Table III lists color arrangements with furniture of other hues. All of the harmonies of Tables II and III satisfy the requirements of high reflectance, low chroma on extended areas, pleasing hues, and harmonious relations

among colors. The 25 suggested arrangements do not, of course, begin to exhaust the possibilities: they merely indicate a few harmonies that are definitely satisfactory for rooms.

TABLE II.—*Some Harmonious Color Arrangements Using Natural-Wood Furniture.*

No.	Ceiling	Wall [a]	Floor	Furniture	Machines	Trim [a]
1	N 9.2	N 9.2	PB 7/2	Unstained wood [b]	N 8	PB 5/6
2	N 9.2	PB 8/2	PB 7/2		N 8	PB 5/6
3	N 9.2	N 9.2	B 7/2		N 8	B 5/6
4	N 9.2	B 8/2	B 7/2		N 8	B 5/6
5	N 9.2	N 9.2	BG 7/2		N 8	BG 5/6
6	N 9.2	BG 8/2	BG 7/2		N 8	BG 5/6
7	N 9.2	N 9.2	G 7/2		N 8	G 5/6
8	N 9.2	G 8/2	G 7/2		N 8	G 5/6
9	N 9.2	Y 9/4 [d]	G 7/2		G 8/2	G 5/6
10	N 9.2	Y 9/4 [d]	PB 7/2		PB 8/2	PB 5/6
11	N 9.2	YR 9/2 [d]	YR 7/2		Y 9/4	YR 5/6

[a] If dado is desired, use these colors for dado and make upper walls white (N 9.2).
[b] Or Y 8/2 or YR 8/2, if natural wood furniture is not used.
[c] Limited to small areas.
[d] Or unstained wood (maple, pine, white harewood, avodire, prima vera, or other wood of high reflectance).

TABLE III.—*Some Harmonious Color Arrangements Using Painted Furniture.*

No.	Ceiling	Wall	Floor	Furniture	Machines	Trim
12	N 9.2	N 9.2	R 7/2	R 8/2	N 8	R 5/6
13	N 9.2	R 8/2	N 7	R 8/2	N 8	R 5/6
14	N 9.2	N 9.2	N 7	R 8/2	N 8	R 5/6
15	N 9.2	R 8/2	R 7/2	R 8/2	PB 8/2	R 5/6
16	N 9.2	N 9.2	GY 7/4	GY 8/4	Y 9/4	GY 5/4
17	N 9.2	GY 9/2	GY 7/4	GY 8/4	Y 9/4	GY 5/4
18	N 9.2	N 9.2	G 7/2	G 8/2	Y 9/4	G 5/6
19	N 9.2	G 8/2	G 7/2	G 8/2	N 8	G 5/6
20	N 9.2	N 9.2	BG 7/2	BG 8/2	Y 9/4	BG 5/6
21	N 9.2	BG 8/2	BG 7/2	BG 8/2	Y 9/4	BG 5/6
22	N 9.2	N 9.2	B 7/2	B 8/2	Y 9/4	B 5/6
23	N 9.2	B 8/2	B 7/2	B 8/2	Y 9/4	B 5/6
24	N 9.2	N 9.2	PB 7/2	PB 8/2	N 8	PB 5/6
25	N 9.2	PB 8/2	PB 7/2	PB 8/2	N 8	PB 5/6

TABLE IV.—*Munsell Values Recommended in Tables II and III.*

	Value
Ceilings	9.2 or above
Walls	8 or 9
Floor	7
Furniture	8
Machines	8 or 9
Trim	5

Chromas for walls, floor, furniture, and machines are 2 in most cases and never exceed 4, in conformity with the principles stated in Section 3. The chroma of the trim is 6 to give a more interesting effect, but these colors must be confined to small areas. Munsell values used in Tables II and III are summarized in Table IV.

In deciding on colors that will harmonize with natural woods, it is necessary to know the approximate Munsell specification of wood. Table V gives the hue, value, and chroma for 35 woods (17). A study of these data shows that the mode is at hue 10YR and chroma 4. Thus a

TABLE V.—*Colors of Natural Woods (Not Stained or Varnished).*

No.	Name	ρ/ρ_B *	Munsell Specification		
			Hue	Value	Chroma
1	White harewood	0.6630	5Y	8	3
2	Avodire	0.5903	5Y	8	4
3	Plain maple	0.5845	2.5Y	8	3
4	Ribbon prima vera	0.5719	2.5Y	8	4
5	Pine	0.5685	2.5Y	8	4
6	Figured aspen	0.5111	2.5Y	7	4
7	Figured capomo	0.5021	2.5Y	7	4
8	Flat-cut prima vera	0.4979	2.5Y	7	4
9	Faux satine	0.4349	2.5Y	7	3
10	Quartered oak	0.4177	10YR	7	4
11	Gray harewood	0.3900	5Y	7	2
12	Flat-cut bella rosa	0.3856	10YR	7	5
13	Flat-cut oak	0.3726	10YR	7	4
14	Birch	0.3709	7.5YR	7	3
15	Satinwood	0.3680	10YR	6	4
16	Zebra wood	0.3643	10YR	6	4
17	Cypress	0.3327	10YR	6	4
18	Mahogany	0.3022	7.5YR	6	5
19	Lacewood	0.2991	7.5YR	6	4
20	Figured red gum	0.2829	10YR	6	3
21	English oak	0.2772	10YR	6	4
22	Tigerwood	0.2762	10YR	6	4
23	Figured teak	0.2668	10YR	6	4
24	Pearwood	0.2512	5YR	5	4
25	Bubinga	0.2505	2.5YR	5	4
26	Figured paldao	0.2496	10YR	5	3
27	Flat-cut walnut	0.2432	7.5YR	5	2
28	Cherry wood	0.2402	5YR	5	4
29	Claro walnut	0.2253	7.5YR	5	3
30	Makori wood	0.2180	5YR	5	4
31	Figured quartered walnut	0.2121	10YR	5	3
32	Goncolo alves	0.2048	7.5YR	5	4
33	Sapeli	0.2032	5YR	5	4
34	Oriental wood	0.1481	10YR	4	2
35	Brazilian rosewood	0.1273	7.5YR	4	2
	Typical wood		10YR	7	4

* Reflectance for incident radiation from Planckian radiator at 2842° K.

representative natural wood may be assumed to have the Munsell specification, YR 7/4. Harmonious hues to be used with wood (Table II) are YR, Y, G, BG, B, and PB. The first two give identity or similarity, while the others give contrast.

Maximum simplicity is obtained with walls of uniform color rather than with a dado of a different color. If, for reasons of maintenance, a gloss paint is used for the dado, it can have the same color as the matt paint employed on the upper part of the wall. Or the colors listed under "wall" in Tables II and III may be obtained with a gloss paint for the dado, the upper walls being white to match the ceiling. The

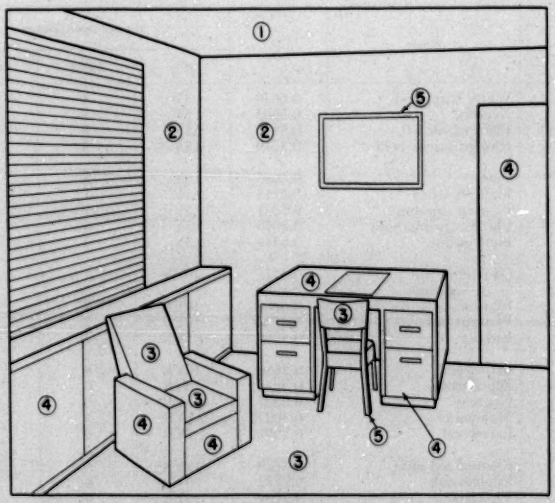

FIG. 4. Representative color arrangement for a room.

1. White (N 9.2)
2. PB 8/2
3. PB 7/2
4. YR 8/2
5. PB 5/6

trim color may be used as a border in the floor, a stripe on furniture or machinery, upholstery on chairs, and similar small areas. Recommended colors for machines are discussed in Section 5. In Table III, the column marked "furniture" applies particularly to desk tops. If steel desks are used, they may be gray (N 8) in all cases, with linoleum tops having the colors designated in the table, or they may be finished in any of the colors used in the room. A representative color arrangement is shown in Fig. 4. The appearance of any of the colors may be obtained by reference to the *Munsell Book of Color* (18).

5. MACHINES.

Typewriters and computing machines have traditionally been finished in black. Factory machinery has likewise been of low reflectance. Manufacturers should be induced to standardize on a set of lighter finishes that will harmonize with modern room colors. A possible set of four is indicated in Table VI. These four combinations are the only

TABLE VI.—*Suggested Colors for Machines.*

	Prominent Parts	Subsidiary Parts	Trim *
A	N 8	N 7	N 5
B	PB 8/2	PB 7/2	PB 5/2
C	G 8/2	G 7/2	G 5/2
D	Y 9/4	YR 8/2	YR 5/2

* White may be used; also black, if confined to very small areas.

ones needed for the 25 rooms of Tables II and III. In fact, the neutral harmony (A) may be used in all cases if desired.

The prominent parts of the machine are finished in the color designated in the second column of Table VI. This corresponds to the "spotlight" or "focal" colors recommended by the paint manufacturers (7). The less important parts of the machine are finished in the

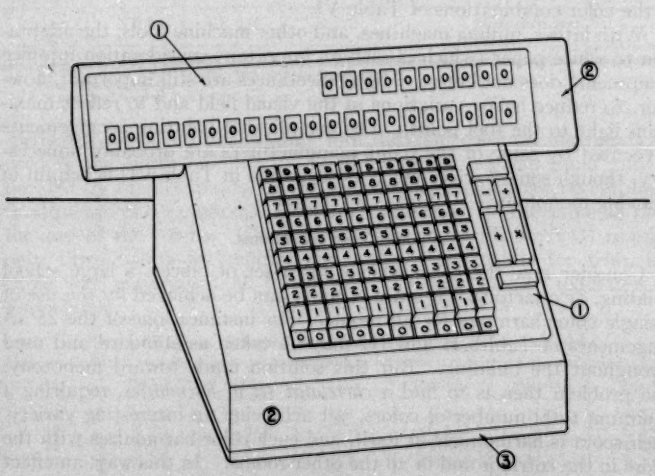

FIG. 5. Representative color arrangement for a computing machine.

1, White (N 9.2)
2, N 8
3, N 5

colors of the third column. Further details, such as stripes, may utilize the colors of the fourth column.

An example is shown in Fig. 5. Some of the latest computing machines are dressed up in very complicated color schemes and even use some surfaces of high reflectance. But they still retain black dials with white figures. This is fundamentally wrong. The eyes of the operator are adapted ordinarily to white paper; but when they are focused on the dials, they will proceed to adapt to the low helios of the black surface (6). This fluctuating state of adaptation, as the operator glances back and forth between paper and dials, constitutes a bad visual condition which is probably responsible for much of the eyestrain associated with the use of a computing machine. The condition would not be so bad if the 30 to 60 seconds required for re-adaptation were available at each movement of the eyes between paper and machine. But this is not the case. Consequently, a visual task where speed and precision are essential must be performed with the eyes imperfectly adapted to the work. Similar conditions exist to some extent with black typewriters, punched-card machines, and other office equipment.

The solution of the difficulty consists in the use of high reflectances and low chromas. In particular, all numbers must be black on white backgrounds. In the machine of Fig. 5, the dials and keys are white with black numerals. The remainder of the machine is finished in any of the color combinations of Table VI.

With lathes, milling machines, and other machine tools, the adaptation to white paper (which is such an important consideration in office equipment) does not enter. High reflectances are still important, however, to reduce helios variations in the visual field and to reflect maximum light to the tool point. The complicated painting arrangements advocated by some of the paint manufacturers are probably unnecessary, though some variety (such as suggested in Table VI) is helpful in relieving monotony.

6. CORRELATED HARMONIES.

Consider now the color scheme for a set of offices, a large school building, or a factory. A sense of unity can be achieved by the use of a single color harmony for all rooms. For instance, one of the 25 arrangements of Tables II and III may be taken as standard and used throughout the building. But this solution tends toward monotony. The problem then is to find a *correlated set of harmonies*, requiring a minimum total number of colors, yet achieving an interesting variety. Each room is harmonious in itself, and each color harmonizes with the colors in the corridor and in all the other rooms. In this way, an effect of unity is maintained, but without monotony.

The theory of color harmony (19) shows that pleasing hue relations, arranged in order of decreasing aesthetic measure, are *identity*, *contrast*,

and *similarity*. There is an advantage, therefore, in employing contrasting hues rather than similar hues. A good compromise between complexity and monotony is achieved by using four different harmonies in the building. One gray harmony may be employed, and the dominant hues of the other three harmonies are made mutually contrasting. Reference to Figs. 2 and 3 shows that there are ten such hue combinations. Elimination of those containing the hues P and RP leaves only 4 possibilities:

(I) N, R, GY, PB;
(II) N, R, GY, B;
(III) N, R, G, PB;
(IV) N, YR, G, PB.

If unstained-wood furniture is used, YR should be taken as one of the dominant hues. This means the use of (IV). A suggested set of room colors is given in Table VII, based on N, YR, G, PB. With this

TABLE VII.—*A Correlated Set of Harmonies Using Natural-Wood Furniture.*

No.	Ceiling	Wall [a]	Floor	Furniture	Machines	Trim [a]
1	N 9.2	N 9.2	PB 7/2	Unstained wood [b]	N 8	PB 5/6
7	N 9.2	N 9.2	G 7/2		N 8	G 5/6
8	N 9.2	G 8/2	G 7/2		N 8	G 5/6
26	N 9.2	YR 9/4 [d]	YR 7/2		N 8	YR 5/6
	N 9.2	YR 9/4	YR 7/2	For corridors		

[a] If dado is desired, use the wall colors for dado and make upper walls white (N 9.2).
[b] Or Y 8/2 or YR 8/2, if natural wood furniture is not used.
[c] Limited to small areas.
[d] Or unstained wood of high reflectance (see Table V).

correlated set of harmonies, only four different color arrangements are used in the entire building. The harmony for a given room may be decided by the occupants, or the four arrangements may be employed in sequence or at random. Corridor colors may be the same as those for any of the rooms. The entire arrangement (Table VII) requires only three paints for ceilings and walls, three paints for trim, and three floor materials. All furniture is the same and all machines are gray (N 8).

TABLE VIII.—*A Correlated Set of Harmonies Using Painted Furniture.*

No.	Ceiling	Wall [a]	Floor	Furniture	Machines	Trim [b]
13	N 9.2	R 8/2	N 7	R 8/2	N 8	R 5/6
19	N 9.2	G 8/2	G 7/2	G 8/2	N 8	G 5/6
24	N 9.2	N 9.2	PB 7/2	PB 8/2	N 8	PB 5/6
27	N 9.2	N 9.2	N 7	PB 8/2	N 8	R 5/6
	N 9.2	G 8/2	G 7/2	For corridors		

[a] If dado is desired, use the wall colors for dado and make upper walls white (N 9.2).
[b] Limited to small areas.

Another suggested set is listed in Table VIII, the hues being N, R, G, PB. Again, three paints suffice for all ceilings and walls and three for trim, while three floor materials are employed. The designer can easily make up similar correlated sets if he is not satisfied with those of Tables VII and VIII.

7. SUMMARY.

The purpose of the paper has been to correlate existing scientific data and methods on light and color in rooms and to apply them to the determination of satisfactory color arrangements. The work is based on

(*a*) The integral-equation method of predetermining helios distributions in rooms,
(*b*) The theory of color harmony.

The results are in approximate agreement with the best of modern practice (20) but differ from previous formulations by considering the effect of lighting and by employing a scientific specification of color instead of a set of trade names or similar vague terminology.

The application of these methods leads to 25 suggested color harmonies for rooms (Tables II and III). Any of these arrangements (used with proper ceiling lighting) will satisfy the 3:1 helios relation and the other objectives mentioned in Section 2. A satisfactory design requires also a consideration of reflectances and colors of furniture and machines. Suggestions are listed in Tables II, III, and VI.

Finally, the question of correlated sets of harmonies is treated. Tables VII and VIII present arrangements that allow an interesting variety of colors in a set of rooms. For any given room, there is a choice of four harmonies, yet the total number of paints and floor materials is kept at a minimum.

REFERENCES.

(1) A. A. BRAINERD, "Achievement of poise in a lighting system," *Illum. Eng.*, Vol. 40, p. 455 (1945);
FABER BIRREN, "Color in the plant," *Factory Management and Maintenance*, Feb. 1945;
D. L. HADLEY AND C. B. RYDER, "Can industrial color finishes be effectively standardized?", *Industrial Standardization*, Vol. 16, p. 151 (1945);
FABER BIRREN, "Functional color," Crimson Press, New York, 1937;
D. B. HARMON, "Lighting and child development," *Illum. Eng.*, Vol. 40, p. 199 (1945);
"The influence of lighting, eyesight, and environment upon work production," Public Bldgs. Administration, Washington, D. C., 1947;
C. D. GIBSON, "Conditioning schoolrooms for visual comfort and efficiency," *Illum. Eng.*, Vol. 42, p. 758 (1947);
W. H. KAHLER AND J. A. MEACHAM, "Correlation of brightness ratios and decoration," *Illum. Eng.*, Vol. 43, p. 175 (1948).

(2) PARRY MOON, "Interflections in finite cylinders," *J. Opt. Soc. Am.*, Vol. 31, p. 223 (1941); "Interflections in lightwells," *Ibid.*, p. 301 (1941); "Interflections in rooms," *Ibid.*, p. 374 (1941);
PARRY MOON AND D. E. SPENCER, "Light distributions in rooms," J. FRANKLIN INST., Vol. 242, p. 111 (1946).

(3) H. L. LOGAN, "Specification points of brightness," *I.E.S. Trans.*, Vol. 34, p. 881 (1939).
(4) I. E. S. Committee on Standards of Quality and Quantity for Interior Illumination, "Brightness and brightness ratios," *Illum. Eng.*, Vol. 39, p. 713 (1944); "Interflectance method of predetermining brightness and brightness ratios," *Ibid.*, Vol. 41, p. 361 (1946); "Brightness distribution in rooms," *Ibid.*, Vol. 42, p. 180 (1947).
(5) GEORGE AINSWORTH, Discussion, *Ibid.*, Vol. 36, p. 1047 (1941).
(6) PARRY MOON AND D. E. SPENCER, "Lighting design," Addison-Wesley Press, Cambridge, Mass., 1948.
(7) A. A. BRAINERD AND M. DENNING, "Improved vision in machine tool operations by color contrasts," *Illum. Eng.*, Vol. 36, p. 1397 (1941).
(8) A. A. BRAINERD AND R. A. MASSEY, "Salvaging waste light," *Ibid.*, Vol. 37, p. 738 (1942).
(9) PARRY MOON AND D. E. SPENCER, "Lighting design by the interflection method," J. FRANKLIN INST., Vol. 242, p. 465 (1946).
(10) PARRY MOON, "Colors of furniture," *J. Opt. Soc. Am.*, Vol. 32, p. 293 (1942); "Reflection factors of some materials used in school rooms," *Ibid.*, p. 243.
(11) "Final report of the O. S. A. Subcommittee on the Spacing of the Munsell Colors," *Ibid.*, Vol. 33, p. 385 (1943).
(12) PARRY MOON AND D. E. SPENCER, "Area in color harmony," *Ibid.*, Vol. 34, p. 93 (1944).
(13) PARRY MOON AND D. E. SPENCER, "A simple criterion for quality in lighting," *Illum. Eng.*, Vol. 42, p. 325 (1947).
(14) D. E. SPENCER, "Classroom lighting," *Ibid.*, Vol. 43, p. 743 (1948).
(15) PARRY MOON AND D. E. SPENCER, "Geometric formulation of classical color harmony," *J. Opt. Soc. Am.*, Vol. 34, p. 46 (1944).
(16) A. H. MUNSELL, "A color notation," Munsell Color Co., Baltimore, 1929.
(17) PARRY MOON AND D. E. SPENCER, "The color of unstained wood," *J. Opt. Soc. Am.*, Vol. 28, p. 405 (1948).
(18) "Munsell book of color," Munsell Color Co., Baltimore, 1942.
(19) PARRY MOON AND D. E. SPENCER, "Aesthetic measure applied to color harmony," *J. Opt. Soc. Am.*, Vol. 34, p. 234 (1944).
(20) PARRY MOON AND D. E. SPENCER, "A survey of colors for industry," to be published.

A Supersonic Parachute (*Science*, Vol. 108, No. 2816).—A supersonic parachute, or "rotochute," which can safely lower instruments from rockets flying as high as 100 miles, has been developed in General Electric's General Engineering and Consulting Laboratory from an invention by I. B. Bensen. Blown free at the peak of a rocket's trajectory, the device can brake the equipment's rate of fall from the original supersonic speed to a landing speed of 27 mph. The most successful model measures about 4 ft. long by 8 in. wide and has a propeller or vanes, about 8 ft. long, which begin to revolve as they fall into denser air until they are horizontal. The high-altitude rockets for which the rotochute is devised are now being fired at White Sands, New Mexico, as part of GE's rocket-research program being conducted by the Army Ordnance Department.

Pilot Plant Production of Equilenin.—A pilot plant to produce equilenin, a hormone that may be convertible into the male and female sex hormones which have proved of value in medical therapy and cancer research, has been set up by the Wisconsin Alumni Research foundation at the University of Wisconsin.

The value of equilenin, Wisconsin scientists say, lies in the fact that it is the only hormone that can be made synthetically from coal tar products.

The process being used at the pilot plant is the result of basic research at the University which began in 1941 under the direction of Prof. William S. Johnson. Drs. Jack W. Petersen and C. David Gutsche, then graduate students in chemistry, were the major collaborators in the project.

The synthesis they developed is elaborate, consisting of some fifteen steps. But yields are good and, most important, the process gives scientists a potentially unlimited source of the hormone from readily available coal tar products.

Although the hormone has no value outside a research laboratory at the present time, scientists at the University plan to investigate the possibility of converting it into hormones of medical importance. For example, it may be possible to convert equilenin into estrone, a female sex hormone, or into a substance closely related to testosterone, the male sex hormone, since all three have basically similar structures.

Estrone and testosterone are being used in studies of the cause and possible cure of cancer. Testosterone has been found to bring relief in many cases to late stages of breast cancer.

Practically no work has been done up to the present time on converting equilenin into these hormones mainly because it has not been available in sufficient quantity. Equilenin, in the past, has been isolated along with the more potent hormone, estrone, from the urine of pregnant mares. Only small amounts are available from this source and it is difficult to separate it from the estrone. The pilot plant should make larger quantities available and pave the way for intensive research.

The plant itself is in a laboratory in the new Wisconsin Alumni Research Foundation building on the Wisconsin campus which has been set aside for special work on long term projects. Dr. Eugene Woroch is in charge of the plant and is assisted by Robert Kloss and Peter Jackson, who received their master's degrees in chemistry from the University of Wisconsin.

THE RELATIVE STOPPING POWER OF GASES FOR ALPHA PARTICLES FROM POLONIUM.

BY

T. N. HATFIELD,[1] A. E. LOCKENVITZ[1] AND M. Y. COLBY.[1]

ABSTRACT.

The relative stopping power for alpha particles from polonium is determined for eighteen different gases, including three groups of isomers. The additive law is shown to hold for the gases for which the stopping power of the constituents is known and the law is used to calculate the stopping power of carbon, sulfur, chlorine, iodine and fluorine from gases containing these elements.

A study of the relative stopping power of gases for alpha particles from polonium is being made and this constitutes a report on measurements to date. The specific ionization apparatus of Colby and Hatfield [2] is used in making the range measurements. This apparatus employs a fixed source-to-electrode distance, and ionization currents are measured as a function of the gas pressure. The general gas law is assumed to hold since the van der Waals corrections for the gases used are small and within the experimental error of pressure measurements.

In determining the relative stopping power of a gas, the question arises as to whether the extrapolated range can be used, particularly if the straggling is large. In the present work there is a straggling due to the geometry of the apparatus in addition to the source straggling. However, the use of extrapolated range to determine relative stopping power is assumed valid if, for a mixture of two gases, the stopping power calculated from the extrapolated range obeys the additive law. For a mixture of two gases containing a fraction F_1 of molecules with stopping power S_1 and a fraction F_2 of molecules with stopping power S_2, we have the additive law:

$$S = F_1S_1 + F_2S_2, \tag{1}$$

where S is the stopping power of the mixture. Taking the stopping power as proportional to the reciprocal of the extrapolated range r, Eq. 1 becomes:

$$\frac{1}{r} = \frac{F_1}{r_1} + \frac{F_2}{r_2}. \tag{2}$$

Since all ionization curves in the present work are obtained with a fixed source-to-electrode distance, the "extrapolated pressure range" p is

[1]Department of Physics, The University of Texas, Austin, Texas.

[2]M. Y. Colby and T. N. Hatfield, *Rev. Sci. Inst.*, Vol. 12, p. 62 (1941).

proportional to the extrapolated range r and Eq. 2 may be written:

$$\frac{1}{p} = \frac{F_1}{p_1} + \frac{F_2}{p_2}. \tag{3}$$

To test the validity of using the extrapolated pressure range with the present apparatus in determining relative stopping powers, mixtures of CO_2 and N_2 were made and the extrapolated pressure ranges determined. Table I gives a comparison of the measured values with

TABLE I.—*Measured and calculated extrapolated pressure ranges (p) of mixtures of CO_2 and N_2.*

Fraction CO_2	Measured p	Calculated p (by Eq. 3)
1.0	17.91	
0.8456	18.92	18.92
0.6754	20.18	20.17
0.4129	22.48	22.47
0.2357	24.27	24.33
0.2184	24.43	24.53
0.0	27.36	

those calculated from Eq. 3. Since the two values are essentially equal, the stopping power can be taken as proportional to the reciprocal of the extrapolated pressure range.

The collimated disc of alpha particles obtained with the apparatus gives an ionization current of the order of 10^{-11} amp. and thus affords an easy method of getting the specific ionization curves. Figure 1 shows

FIG. 1. Diagram of ionization chamber.

A—Collimated disc of alpha particles
C—Collimating plates
E—Collecting electrode
S—Alpha particle source

the general plan of the apparatus which is completely described in the previous reference. At present a d-c. amplifier and microammeter are used in place of the original compensator and electrometer for measuring the ionization currents. The amplifier is a revision of the 100 per cent feed-back amplifier designed by Rense and Heidelberg [3] and has proven to be quite satisfactory. The polonium alpha particle source is deposited on a silver ring which has been evaporated onto the 6-mm. glass tubing at the center of the chamber. The strength of the source is of the order of 10^4 alpha particles per sq. mm. per sec., giving 4×10^3 alpha particles per sec. in the collimated disc.

The ionization currents measured at different gas pressures are cor-

[3] Q. S. HEIDELBERG and W. A. RENSE, *Rev. Sci. Inst.*, Vol. 11, p. 386 (1940).

rected to standard pressure, giving the ionization current which would be produced if the mass of the gas above the collecting electrode were constant. The pressures and currents are all corrected to one temperature if the temperature of the gas changes during one complete measurement. The ionization currents are plotted against the pressures, giving a curve as shown in Fig. 2. The extrapolated pressure range is obtained

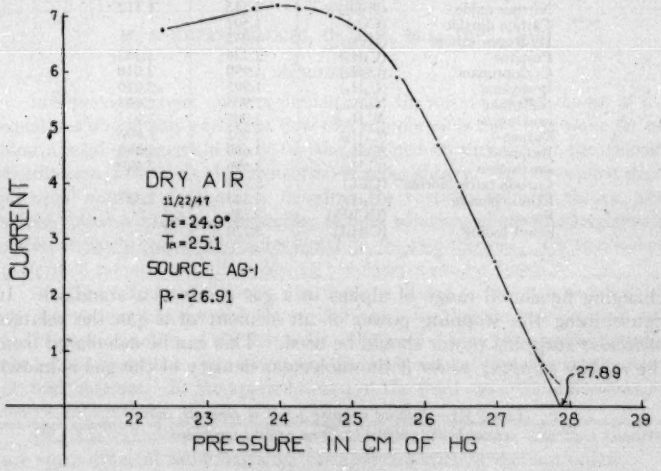

FIG. 2.

by extrapolating to zero the straight line portion of this ionization curve. This extrapolated pressure range is then corrected to 15° C. The ratio of the extrapolated pressure range for dry air to that for a certain gas is the relative stopping power of that gas. The precision in reproducing these extrapolated pressure ranges is of the order of half a per cent.

The results on gases measured to date are given in Table II. The calculated relative stopping power on the basis of the additive law is also given for compounds for which the stopping powers of the constituents are known.

In Table III are given elements for which the stopping power is calculated from the stopping power of compounds containing the elements.

The ratio of the extrapolated pressure range for air to that for a given gas has been taken as the *relative stopping power* of the gas. This stopping power is the one that should be used in identifying the gases, determining the per cent composition of a mixture of two gases, or

TABLE II.—*The relative stopping power of gases measured.*

Gas		Relative Stopping Power	
		Experimental	From additive law
Oxygen	(O_2)	1.055	
Hydrogen	(H_2)	0.223	
Nitrogen	(N_2)	0.984	
Nitrous oxide	(N_2O)	1.515	1.512
Carbon dioxide	(CO_2)	1.502	
Hydrogen sulfide	(H_2S)	1.095	
Propane	(C_3H_8)	2.224	2.233
Cyclopropane	(C_3H_6)	1.990	2.010
Propylene	(C_3H_6)	1.995	2.010
N-Butane	(C_4H_{10})	2.905	2.913
iso-Butane	(C_4H_{10})	2.907	2.913
Butene-1	(C_4H_8)	2.691	2.690
Butene-2	(C_4H_8)	2.690	2.690
iso-Butene	(C_4H_8)	2.696	2.690
Carbon tetrachloride	(CCl_4)	3.984	
Ethyl chloride	(C_2H_5Cl)	2.343	2.337
Freon	(CCl_2F_2)	3.259	
Ethyl Iodide	(C_2H_5I)	3.129	

changing measured range of alphas in a gas to air as a standard. In determining the stopping power of an element of a gas the relative *molecular* stopping power should be used. This can be calculated from the *relative stopping power* if the molecular density of the gas is known

TABLE III.—*Relative stopping power of elements, calculated by the additive law from gaseous compounds.*

Element	Relative Stopping Power	Calculated from:
Carbon	0.447	CO_2 and O_2
Sulfur	0.872	H_2S and H_2
Chlorine	0.884	CCl_4 and C
Iodine	1.678	C_2H_5I, C and H_2
Fluorine	0.524	CCl_2F_2, C and Cl_2

at the pressures and temperatures at which the measurements are made. The two stopping powers are identical if the gas has the same number of molecules per unit volume as air.

ON THE TWO-DIMENSIONAL STEADY TURBULENT FLOW OF A COMPRESSIBLE FLUID FAR BEHIND A SOLID SYMMETRICAL BODY. II. (VORTICITY TRANSFER THEORY).[1]

BY

M. Z. KRZYWOBLOCKI, Dr. Ae.E., M.A., DIPL. ING.[2]

INTRODUCTION.

In a previous paper under a similar title [3] the author solved the set of five equations of a steady turbulent flow of a compressible fluid in a wake far behind a solid symmetrical body for the assumed condition that the velocity distribution is based on the momentum transfer theory. In the present paper an equal number of equations, based on the vorticity transfer theory, were solved. Comparison and inspection of both solutions shows the largeness of discrepancies between the results based on the two theories. On the basis of performed calculations, the following conclusions can be drawn:

(*a*). The calculated velocity distributions, based on the two theories, i.e., the momentum transfer and the vorticity transfer theories, are identical only to the approximation of the first order. Beginning from the approximation of the second order there are increasing discrepancies between the results based on both theories. In the approximation of the third order the differences are very significant.

(*b*). The calculated temperature distributions, based on the two theories, are much different beginning from the approximation of the first order.

(*c*). The calculated pressure and density distributions, based on the two theories, are different beginning from the first approximation, giving results different from zero, i.e., from the approximation of the second order.

I. THE GENERAL EQUATIONS OF MOTION AND BASIC LAWS.[4]

The equations of motion are derived under the assumption that motion is steady and two-dimensional. All the terms containing the viscosity coefficient are neglected. The x-axis is assumed to be vertical and no extraneous forces are acting on the fluid. In the equation of energy all the terms containing the

[1] Presented at the Meeting of the American Mathematical Society, Applied Mathematics Section, St. Louis, November 28–29, 1947.

[2] Associate Professor, Department of Aeronautical Engineering, University of Illinois, Urbana, Ill.

[3] The author presumes that the reader is acquainted with the author's previous paper, "On the Two-Dimensional Steady Turbulent Flow of a Compressible Fluid Far Behind a Solid Symmetrical Body. I. (Momentum Transfer Theory)," JOURNAL OF THE FRANKLIN INSTITUTE, Vol. 247, No. 1, 1949, p. 33.

[4] All the symbols and notations, used in the present paper, are the same as those used in the author's previous paper, *loc. cit.* The numbers of equations in the present paper are all with "prime" (1′).

viscosity and thermal conductivity coefficients are neglected. Under these assumptions we have the following forms of equations:
two equations of motion:

$$\rho[\partial_x(\tfrac{1}{2}\vec{V}^2) - vz] = -\partial_x p, \tag{1'}$$

$$\rho[\partial_y(\tfrac{1}{2}\vec{V}^2) + uz] = -\partial_y p, \tag{1'a}$$

the equation of continuity:

$$\partial_x(\rho u) + \partial_y(\rho v) = 0. \tag{2'}$$

Making use of the equation of continuity multiplied by JTc_p permits writing the equation of conservation of energy in the form:

$$Tc_p[\partial_x(u\rho T) + \partial_y(v\rho T)] + p(\partial_x u + \partial_y v) = 0. \tag{3'}$$[5]

The equation of state for ideal gases is:

$$p = R\rho T. \tag{4'}$$

In the equations cited above, z is the z-component of the vorticity vector and u and v are the components of the velocity vector:

$$\vec{V} = \vec{i}u + \vec{j}v, \qquad z = \partial_x v - \partial_y u. \tag{4'a}$$

As is known, in the vorticity transfer theory the assumption is made that all the velocity components, the density, the temperature and the pressure are subject to fluctuations. Let us substitute into the equations given above the values:

$$u = \bar{u} + u', \qquad v = \bar{v} + v', \qquad z = \bar{z} + z', \tag{5'}$$
$$p = \bar{p} + p', \qquad \rho = \bar{\rho} + \rho',$$

and let us apply the averaging process. The result is:
first equation of motion:

$$\begin{aligned}\bar{\rho}(\bar{u}\partial_x\bar{u} + \bar{v}\partial_y\bar{u}) + \bar{\rho}(\overline{u'\partial_x u'} + \overline{v'\partial_x v'}) \\ + \bar{u}\,\overline{\rho'\partial_x u'} + \bar{v}\,\overline{\rho'\partial_x v'} + \overline{u'\rho'}\partial_x\bar{u} + \overline{v'\rho'}\partial_y\bar{u} \\ + \overline{u'\rho'\partial_x u'} + \overline{v'\rho'\partial_x v'} - \bar{\rho}\,\overline{v'z'} - \bar{v}\,\overline{\rho'z'} - \overline{v'\rho'z'} + \partial_x\bar{p} = 0,\end{aligned} \tag{6'a}$$

second equation of motion:

$$\begin{aligned}\bar{\rho}(\bar{u}\partial_x\bar{v} + \bar{v}\partial_y\bar{v}) + \bar{\rho}(\overline{u'\partial_y u'} + \overline{v'\partial_y v'}) \\ + \bar{u}\,\overline{\rho'\partial_y u'} + \bar{v}\,\overline{\rho'\partial_y v'} + \overline{u'\rho'}\partial_x\bar{v} + \overline{v'\rho'}\partial_y\bar{v} \\ + \overline{u'\rho'\partial_y u'} + \overline{v'\rho'\partial_y v'} + \bar{\rho}\,\overline{u'z'} + \bar{u}\,\overline{\rho'z'} + \overline{u'\rho'z'} + \partial_y\bar{p} = 0,\end{aligned} \tag{6'b}$$

continuity equation of a fictitious mean motion with the mean density $\bar{\rho}$:

$$\partial_x(\bar{u}\bar{\rho}) + \partial_y(\bar{v}\bar{\rho}) = 0, \tag{7'}$$

equation of state:

$$\bar{p} = R(\bar{\rho}\bar{T}), \tag{8'}$$

[5] In its final form this equation will not be changed as compared with the equation of energy in the previous paper, *loc. cit.*

equation of energy:

$$\mathfrak{T}c_p(\bar{K} + K' + L) + N = 0, \tag{9'}$$

$$\bar{K} = \partial_x(\bar{u}\bar{\rho}\bar{T}) + \partial_y(\bar{v}\bar{\rho}\bar{T}), \tag{9'a}$$

$$K' = \partial_x(\overline{u'\rho'T'}) + \partial_y(\overline{v'\rho'T'}), \tag{9'b}$$

$$L = \partial_x(\bar{u}\,\overline{\rho'T'} + \bar{\rho}\,\overline{u'T'} + \bar{T}\,\overline{u'\rho'}) + \partial_y(\bar{v}\,\overline{\rho'T'} + \bar{\rho}\,\overline{v'T'} + \bar{T}\,\overline{v'\rho'}), \tag{9'c}$$

$$N = \bar{p}(\partial_x\bar{u} + \partial_y\bar{v}) + \overline{p'\partial_x u'} + \overline{p'\partial_y v'}, \qquad \overline{\rho'T'} = 0. \tag{9'd}$$

On the basis of the assumption that the turbulent [6] motion is confined to the plane xy, or on the basis of the modified vorticity transport theory [7] we may write:

$$\overline{u'z'} = \overline{v'z'} = -\overline{l_T v'}(\partial_x\bar{z} + \partial_y\bar{z})$$
$$= l_T^2\partial_y\bar{u}(\partial_{xy}^2\bar{u} + \partial_{yy}^2\bar{v} - \partial_{xx}^2\bar{v} - \partial_{xy}^2\bar{v}) \text{ with } \bar{z} = (\partial_x\bar{v} - \partial_y\bar{u}). \tag{10'a}$$

In all the terms we omit the signs of the absolute value, assuming that the calculations refer to $y > 0$. We also assume:

$$\overline{\rho'z'} = l_T^2\partial_y\bar{\rho}(\partial_{xy}^2\bar{u} + \partial_{yy}^2\bar{v} - \partial_{xx}^2\bar{v} - \partial_{xy}^2\bar{v}), \tag{10'b}$$

$$\overline{u'\rho'z'} = \overline{v'\rho'z'} = l_T^3\partial_y\bar{u}\partial_y\bar{\rho}(\partial_{xy}^2\bar{u} + \partial_{yy}^2\bar{v} - \partial_{xx}^2\bar{v} - \partial_{xy}^2\bar{v}). \tag{10'c}$$

All other terms will be transformed in the same way as in the author's previous paper:

$$\bar{\rho}\,\overline{u'\partial_x u'} = \bar{\rho}\,\overline{v'\partial_x v'} = -l_T\bar{\rho}\partial_y\bar{u}\partial_x(l_T\partial_y\bar{u}), \tag{11'a}$$

$$\bar{u}\,\overline{\rho'\partial_x u'} = -l_T\bar{u}\partial_y\bar{\rho}\partial_x(l_T\partial_y\bar{u}), \tag{11'b}$$

$$\bar{v}\,\overline{\rho'\partial_x v'} = -l_T\bar{v}\partial_y\bar{\rho}\partial_x(l_T\partial_y\bar{u}), \text{ etc.,} \tag{11'c}$$

$$\overline{u'\rho'}\partial_x\bar{v} = -l_T^2\partial_y\bar{u}\partial_x\bar{v}\partial_y\bar{\rho}, \tag{11'd}$$

$$\overline{u'\rho'\partial_y u'} = \overline{v'\rho'\partial_y v'} = -l_T^3\partial_y\bar{u}\partial_{yy}^2\bar{u}\partial_y\bar{\rho}, \text{ etc.,} \tag{11'e}$$

$$p' = l_T\partial_y\bar{p} = 0, \tag{11'f}$$

[6] The following papers were used as references:

G. I. TAYLOR, "The Transport of Vorticity and Heat Through Fluids in Turbulent Motion," *Proc.* Roy. Soc. London, Series A, Vol. 135, 1932, p. 685.

G. I. TAYLOR, "Distribution of Velocity and Temperature Between Concentric Rotating Cylinders," *Ibid.*, Series A, Vol. 151, 1935, p. 494.

S. GOLDSTEIN, "The Similarity Theory of Turbulence and Flow Between Parallel Planes and Through Pipes," *Ibid.*, Series A, Vol. 159, 1937, p. 473.

G. I. TAYLOR, "Flow in Pipes and Between Parallel Planes," *Ibid.*, Series A, Vol. 159, 1937, p. 496.

"Modern Developments in Fluid Dynamics," edited by S. Goldstein, Oxford, Clarendon Press, 1938.

G. N. ABRAMOVICH, "The Theory of a Free Jet of a Compressible Gas," Report No. 377, Central Aero-Hydrodynamical Institute, Moscow, 1939, or N.A.C.A. Technical Memorandum, No. 1058, March, 1944.

[7] See papers of Taylor, *Proc. Roy. Soc.*, Vol. 151 (1935), p. 945, Eqs. (3) and (9); *Ibid.*, Vol. 159 (1937), p. 500, Eq. (5), and other references. The subscript T in the term l_T refers to the name of Taylor, the author of the vorticity transport theory.

$$\overline{p'\partial_x u'} = -l_T\partial_x(l_T\partial_y\bar{u})\partial_y\bar{p}, \tag{11'g}$$

$$\overline{p'\partial_y v'} = -l_T^2\partial_{yy}^2\bar{u}\partial_y\bar{p}, \text{ etc.} \tag{11'h}$$

In the terms given above the mixing length l_T is assumed to be the same, since the same mechanism is at work. Similarly, as in the previous paper, we assumed that the length l_T is a function of x only and not of y. As is known, there exists the following relation between the values of the mixing lengths, as given by Prandtl (l) and Taylor (l_T) relatively: [8]

$$l_T^2 = 2l^2. \tag{12'}$$

This formula will be used in all the expressions. Substituting all the terms of (11') into the general equations and omitting the bars over all the letters we obtain the following set of equations:

two equations of motion:

$$\rho(u\partial_x u + v\partial_y u) = \sum_{i=1}^{3} F_i + \partial_x p = 0, \tag{13'}$$

$$F_1 = 2l\{2\rho\partial_y u + [(u+v) + 2^{\frac{1}{2}}l\partial_y u]\partial_y\rho\}\partial_x(l\partial_y u), \tag{13'a}$$

$$F_2 = 2l^2\partial_y u\partial_y\rho(\partial_x u + \partial_y u), \tag{13'b}$$

$$F_3 = 2l^2[\rho\partial_y u + (2^{\frac{1}{2}}l\partial_y u + v)\partial_y\rho](\partial_{xy}^2 u + \partial_{yy}^2 u - \partial_{xx}^2 v - \partial_{xy}^2 v), \tag{13'c}$$

$$\rho(u\partial_x v + v\partial_y v) - \sum_{i=4}^{6} F_i + \partial_y p = 0, \tag{14'}$$

$$F_4 = 2l^2\{2\rho\partial_y u + [(u+v) + 2^{\frac{1}{2}}l\partial_y u]\partial_y\rho\}\partial_{yy}^2 u, \tag{14'a}$$

$$F_5 = 2l^2\partial_y u\partial_y\rho(\partial_x v + \partial_y v), \tag{14'b}$$

$$F_6 = (-2)l^2[\rho\partial_y u + (2^{\frac{1}{2}}l\partial_y u + u)\partial_y\rho](\partial_{xy}^2 u + \partial_{yy}^2 u - \partial_{xx}^2 v - \partial_{xy}^2 v), \tag{14'c}$$

equation of continuity for the mean motion:

$$\partial_x(\rho u) + \partial_y(\rho v) = 0, \tag{15'}$$

equation of energy:

$$p(\partial_x u + \partial_y v) - 2l[\partial_x(l\partial_y u) + l\partial_{yy}^2 u]\partial_y p + \mathcal{T}c_p[\rho(u\partial_x T + v\partial_y T) - (\partial_x\Phi_4 + \partial_y\Phi_5)] = 0, \tag{16'}$$

$$\Phi_4 = 2l^2[(u + 2^{\frac{1}{2}}l\partial_y u)\partial_y\rho\partial_y T + \partial_y(\rho T)\partial_y u], \tag{16'a}$$

$$\Phi_5 = 2l^2[(v + 2^{\frac{1}{2}}l\partial_y u)\partial_y\rho\partial_y T + \partial_y(\rho T)\partial_y u], \tag{16'b}$$

equation of state:

$$p - R(\rho T) = 0. \tag{17'}$$

II. FIRST APPROXIMATION.

1. *The Longitudinal Velocity Component.*

Let us put into the first equation of motion the values:

$$u = u_0 - u_1, \quad v = 0, \quad \rho = \rho_0, \quad p = p_0, \quad l = cax^{\frac{1}{2}}, \quad u_1 \sim x^{\frac{1}{2}}, \tag{18'}$$

[8] See Abramovich, *loc. cit.*, pp. 10, 22, 23.

and keeping only the terms of the order $x'^{-\frac{1}{2}}$, we obtain:

$$u_0\partial_{x'}u_1 + 2c^2x'\partial_{y'}u_1\partial^2_{y'y'}u_1 = 0. \tag{19'}$$

This is an equation exactly identical to the corresponding Eq. (42b) in the previous paper. Introducing the expression

$$u_1 = x'^{-\frac{1}{2}}u_0f_v, \tag{20'}$$

we obtain the same solution as in the previous paper (Eq. 47), if instead of f we write f_v. All the considerations referring to the boundary conditions, to the breadth of the wake, etc., are also valid in the present case.

2. The Transverse Velocity Component.

One may easily check that the result is the same as in the previous paper. In the present paper we shall use the notation v_{v1}.

3. The Temperature.

Let us put into the equation of energy the values:

$$\begin{aligned} &u = u_0 - u_1, \quad v = v_{v1}, \quad p = p_0, \quad \rho = \rho_0, \\ &l = cax'^{\frac{1}{2}}, \quad T = T_0 - T_1, \quad T_1 = T_0x'^{-\frac{1}{2}}\theta_{v1}. \end{aligned} \tag{21'}$$

Keeping only the terms of the order $x'^{-\frac{1}{2}}$ gives the formula:

$$u_0\partial_{x'}T_1 + 2c^2x'(\partial_{y'}u_1\partial^2_{y'y'}T_1 + \partial^2_{y'y'}u_1\partial_{y'}T_1) = 0,^* \tag{22'}$$

in which use was made of the equation of continuity with $\rho_0 = \text{const.}$, i.e. $\partial_x u + \partial_y v = 0$. Equation (22′) is different from the corresponding Eq. (56) of the previous paper because of the presence of the coefficient 2 in the second term. After some easy transformations we obtain

$$\theta_{v1} + \eta\theta'_{v1} - 4c^2(f_v'\theta''_{v1} + f''_v\theta'_{v1}) = 0. \tag{23'}$$

Introducing the variable η_1 we obtain

$$\theta_{v1} + \eta_1\theta'_{v1} - 2(f_v'\theta''_{v1} + f''_v\theta'_{v1}) = 0, \tag{24'}$$

where differentiation is performed with respect to η_1. From (24′) one obtains

$$\eta_1\theta_{v1} - 2(f_v'\theta'_{v1}) = \text{const.} \tag{25'}$$

From the boundary condition that for $\eta_1 = 0$ also $\theta'_{v1} = 0$, we obtain const. $= 0$. Separation of the variables gives the solution:

$$\begin{aligned} &\theta_{v1}(\eta) = (2c^2)^{-\frac{1}{2}}C'(\tfrac{1}{3}\eta^{\frac{3}{2}} - A), \quad \theta_{v1}(\eta_1) = C'(\tfrac{1}{3}\eta_1^{\frac{3}{2}} - A_1), \\ &\theta_{v1}(\xi) = C'A_1(\xi^{\frac{3}{2}} - 1), \quad \theta_{v1}(z) = C'A_1(z - 1). \end{aligned} \tag{26'}$$

$$\begin{aligned} \text{B.C. } &\theta'_{v1} = 0 \quad \text{for} \quad \eta_1 = 0, \\ &\theta_{v1} = 0 \quad \text{for} \quad \eta_1 = \eta_{1\,v1}, \\ &\theta'_{v1} = \tfrac{1}{2}C'(3A_1)^{\frac{1}{2}} \neq 0 \quad \text{for} \quad \eta_1 = \eta_{1\,v1}. \end{aligned} \tag{26'a}$$

* This result is in full accordance with the result obtained by Abramovich, *loc. cit.*, Eqs. (28) and (28a).

The last result represents the error of this approximation.[9] The constant C' must be determined from tests.

4. The Pressure.

Let us substitute into the second equation of motion the values:

$$u = u_0 - u_1, \qquad v = v_{v1}, \qquad p = p_0 - p_1, \qquad \rho = \rho_0. \tag{27'}$$

Keeping only the terms of the order $x'^{-\frac{1}{2}}$, we easily obtain:

$$\partial_{y'} p_1 + 6c^2 x' \rho_0 \partial_{y'} u_1 \partial^2_{y'y'} u_1 = 0. \tag{28'}$$

With the exception of the constant coefficient (6 instead of 2) this equation is similar to Eq. (63) in the previous paper. The same reasoning as in the previous paper will lead to the conclusion that $p_1 = 0$.

5. The Density.

From the equation of state with $\rho = \rho_0 - \rho_1$ and with other values from the first approximation, we easily obtain:

$$\rho_0 T_1 + \rho_1 (T_0 - T_1) = 0. \tag{29'}$$

Assuming that $\rho_1 \sim x'^{-\frac{1}{2}}$, neglecting the possible initial solution resulting from preserving only the terms of the order $x'^{-\frac{1}{2}}$, and keeping only the terms of the order x'^{-1}, gives a formula similar to the formula in the previous paper:

$$\rho_1 T_1 = 0. \tag{30'}$$

The result, corresponding to Eq. (68) is:

$$R_{v1} = 0. \tag{31'}$$

Considerations identical to those given in the previous paper lead to the final result that $R_{v1} = \rho_1 = 0$.

III. SECOND APPROXIMATION.

1. The Longitudinal Velocity Component.

Introducing into the first equation of motion the values:

$$u = u_0 - u_1 - u_2, \qquad v = v_{v1}, \qquad \rho = \rho_0, \qquad p = p_0,$$
$$u_2 = u_0 x'^{-1} g_v(\eta), \qquad l = ac(x'^{\frac{1}{2}} + c_1), \tag{32'}$$

[9] The result obtained for ϑ_{v1} is in full accordance with the result cited by Goldstein ("Modern Developments in Fluid Dynamics," edited by S. Goldstein, Oxford, Clarendon Press, 1938), p. 664, Eq. (37). From the diagram (Fig. 252) in Goldstein's book it is also quite obvious that the curve for ϑ_{v1} does not fit the experimental curve well at the edge of the wake. The experimental curve fulfills the condition that the derivative should vanish at the edge of the wake.

and keeping only the terms of the order x'^{-2}, we easily obtain:

$$u_1\partial_{x'}u_1 - u_0\partial_{x'}u_2 - v_{v1}\partial_{y'}u_1 - 2c^2(\partial_{y'}u_1)^2 - 2c^2x'(3\partial_{y'}u_1\partial^2_{x'y'}u_1 + \partial_{y'}u_1\partial^2_{y'y'}u_2 + \partial^2_{y'y'}u_1\partial_{y'}u_2) - 4c^2c_1x'^{\frac{1}{2}}\partial_{y'}u_1\partial^2_{y'y'}u_1 = 0. \quad (33')$$

Introducing the variable η we obtain:

$$2g_v + \eta g_v' - 4c^2(f_v'g_v'' + f_v''g_v' - 2f_v'^2) - f_vf_v' + 2c^2f_vf_v''(3\eta - 4c_1) = 0. \quad (34')$$

Introducing the variable η_1 we obtain:

$$2g_{v1} + \eta_1g_{v1}' - 2(f_v'g_{v1}'' + f_v''g_{v1}') - f_vf_v' + (2c^2)^{\frac{1}{2}}(4f_v'^2 + 3\eta_1f_v'f_v'') - 4c_1f_v'f_v'' = 0. \quad (35')$$

The subscript "1" in g_{v1} means the function of η_1. Introducing the variable z we obtain:

$$\tfrac{3}{4}z(1-z)d^2_{zz}g_{v1} + \tfrac{1}{2}(1-z)d_zg_{v1} + g_{v1} - \tfrac{1}{2}A_1^4(z-1)^4 - A_1^2c_1(z-1)(4z-1) + \tfrac{1}{4}A_1^2(3A)^{\frac{1}{2}}z^{\frac{1}{2}}(20z^2 - 31z + 11) = 0. \quad (36')$$

The only difference between Eqs. (36′) and (74) is in the form of the last term. This corresponds to the last particular solution of the equation. We have to find it:

$$L(g_{v4}) + \beta_1z^{\frac{1}{2}}(20z^2 - 31z + 11) = 0, \quad (37')$$

$$\beta_1 = \tfrac{1}{4}A_1^2(3A)^{\frac{1}{2}}, \quad (37'a)$$

$$L(g_{v4}) = \tfrac{3}{4}z(1-z)g_{v4}'' + \tfrac{1}{2}(1-z)g_{v4}' + g_{v4}. \quad (37'b)$$

The solution for the unit circle around $z = 0$ is:

$$g_{v41} = a_0 + a_2z + a_3z^{5/2} + a_4z^2 + a_5z^{5/2} + \cdots, \quad (38')$$

The coefficient a_0 must be determined, as previously, from Eq. (86) with the new values of the coefficients c_i, $i \geqslant 2$, in $C(g_4)$, Eq. (86a). The boundary condition: $g_{v41} = 0$ for $z = 0$ is fulfilled. Around $z = 1$ we obtain:

$$g_{v42} = c_2(z-1)^2 + c_3(z-1)^3 + \cdots, \quad (39')$$

Both boundary conditions, i.e. $g_{v42} = 0$ and $g_{v42}' = 0$ for $\xi = \xi_{v1} = 1$ are fulfilled. The second approximation edge curve will be given again by Eq. (90) with the new value of c_2. The other boundary condition of the second kind, i.e., that $(u_1 + u_2)$ should vanish for $\xi = \xi_{v2}$ is not perfectly fulfilled, and the same considerations as in the previous paper show that the first non-vanishing term is of the order $x'^{-\frac{1}{2}}$. Below we shall use the notation: $g_v = g_{v3i} + g_{v3i} + g_{v4i}$, $i = 1,2$.

2. The Transverse Velocity Component.

This will be given identically by the formulas of the previous paper. The only difference is that the new values of the coefficients c_i, $i \geqslant 2$, must be used. The symbol v_{v2} will be used.

3. The Temperature.

Substitute into the equation of energy the values:

$$u = u_0 - u_1 - u_2, \qquad v = v_{v1} + v_{v2}, \qquad \rho = \rho_0, \qquad p = p_0, \tag{40'}$$
$$T = T_0 - T_1 - T_2, \qquad T_2 = T_0 x'^{-1}\theta_{v2},$$

and keep only the terms of the order x'^{-2}. The use of the result from the second approximation to v was made, namely, that $\partial_y v_2 - \partial_x u_2 = 0$. After all the necessary transformations and introducing the variable η_1, we obtain:

$$f_v'\theta_{v2}'' + (f_v'' - 2\eta_1)\theta_{v2}' - 2\theta_{v2} + f_v\theta_{v1} - (2c^2)^{\frac{1}{2}}[2f_v'\theta_{v1}' + \eta_1(f_v''\theta_{v1}' + f_v'\theta_{v1}'')]$$
$$+ g_v'\theta_{v1}'' + g_v''\theta_{v1}' + 4c_1(f_v''\theta_{v1}' + f_v'\theta_{v1}'') = 0. \tag{41'}$$

Introducing the variable z and keeping in mind that $\theta_{v1}'' = 0$, we easily obtain:

$$6z(z-1)\theta_{v2}'' - (14z+4)\theta_{v2}' - 16\theta_{v2} + 8f_v\theta_{v1}$$
$$- 3A_1^{-2}(3A)^{\frac{1}{2}}z^{\frac{1}{2}}(2f_v'\theta_{v1}' + zf_v''\theta_{v1}') + A_1^{-2}(2g_v'\theta_{v1}' + 3zg_v''\theta_{v1}')$$
$$+ 4c_1A_1^{-2}(2f_v'\theta_{v1}' + 3zf_v''\theta_{v1}') = 0. \tag{42'}$$

The homogeneous equation has the same solution as in the previous paper. To find the particular solutions, we have to solve the following set of equations:

$$L(\theta_{v21}) + 8f_v\theta_{v1} = 0, \tag{43'a}$$

$$L(\theta_{v22}) + 4A_1^{-2}c_1(2f_v'\theta_{v1}' + 3zf_v''\theta_{v1}') = 0, \tag{43'b}$$

$$L(\theta_{v23}) - 3(3A)^{\frac{1}{2}}A_1^{-2}z^{\frac{1}{2}}(2f_v'\theta_{v1}' + zf_v''\theta_{v1}') = 0, \tag{43'c}$$

$$L(\theta_{v24}) + A_1^{-2}(2g_v'\theta_{v1}' + 3z\theta_{v1}'g_v'') = 0, \tag{43'd}$$

$$L(\theta_{v2i}) = 6z(z-1)\theta_{v2}'' - (14z+4)\theta_{v2}' - 16\theta_{v2}, \qquad i = 1 \text{ to } 4.^{**} \tag{43'e}$$

The following remarks refer to particular functions:

(θ_{v21}): B.C. $\theta_{v21}' = 0$ to $\eta_1 = 0$. For $\xi = \xi_{v2}$ it must be:

$$T_1(\xi_{v2}) + T_2(\xi_{v2}) = 0. \tag{44'}$$

This condition gives the value of m_0.
The last B.C. is:

$$T_1'(y_{v2}') + T_2'(y_{v2}') = 0 \tag{45'a}$$

$$T_1'(y_{v2}') = 1.5A_1(3A)^{-\frac{1}{2}}C'T_0x'^{-1}(1 + Kx'^{-\frac{1}{2}})^{\frac{1}{2}}, \tag{45'b}$$

$$T_2'(y_{v2}') = (3A)^{-\frac{1}{2}}T_0x'^{-\frac{1}{2}}\sum_{i=1}^{4}\theta_{v2i}'(\xi)\,|_{\xi=\xi_{v2}}, \tag{45'c}$$

$$\theta_{v21}'(\xi_{v2}) = \theta_{v21}'(1) + \cdots = 1.5m_1 + \cdots = \text{const.} \tag{45'd}$$

Hence the first non-vanishing term in T_1' is of the order x'^{-1} and in T_2' of the order $x'^{-\frac{1}{2}}$.

(θ_{v23}): F_4 cannot be determined from the series expansion and must be determined from B.C.

** See Appendix.

(θ_{v24}): The coefficients Ni are given in the previous paper. The values of the coefficiencts Mi are changed. B.C. is fulfilled.

(δ_{v24}): The values of the coefficients are given in the previous paper. Coefficients $K_i - s$ have values different from those obtained in the previous paper. The reasoning concerning the boundary conditions follows the same pattern as in the previous paper.

4. The Pressure.

Put into the second equation of motion the values:

$$u = u_0 - u_1 - u_2, \qquad v = v_{v1} + v_{v2}, \qquad \rho = \rho_0, \qquad p = p_0 - p_2, \quad (46)$$
$$p_2 = p_0 x'^{-\frac{1}{2}} \Pi_{v2},$$

and keep only the terms of the order x'^{-2}. Obtain:

$$\partial_{y'} p_2 = u_0 \rho_0 \partial_{x'} v_1 - 2\rho_0 c^2 [x'(\partial_{y'} u_1 \partial^2_{y'y'} u_2 + \partial^2_{y'y'} u_1 \partial_{y'} u_2) - \partial_{y'} u_1 \partial^2_{x'y'} u_1 + 2c_1 x'^{\frac{1}{2}} \partial_{y'} u_1 \partial^2_{y'y'} u_1]. \quad (47)$$

The solution around $z = 0$ will be the same as in the previous paper and the only changes will be in four coefficients, all others being unchanged. The changed coefficients are given in the Appendix. Similarly as before, the curve has a cusp for $\eta_1 = 0$. The solution around $z = 1$ will be the same as in the previous paper. The following changes must be made as concerning the values of the coefficients: M_1', M_2', will be the same as before;

in M_3' instead of $7.5(3A)^{-\frac{1}{2}} A_1^4 c^2$, put $(-13.5)(3A)^{-\frac{1}{2}} A_1^4 c^2$,
in M_4' instead of $7(3A)^{-\frac{1}{2}} A_1^4 c^2$, put $(-9)(3A)^{-\frac{1}{2}} A_1^4 c^2$,
in M_5' instead of $(\frac{9}{4})(3A)^{-\frac{1}{2}} A_1^4 c^2$, put $(-2)(3A)^{-\frac{1}{2}} A_1^4 c^2$, etc.

All B.C. will remain the same as in the previous paper.

5. The Density.

Put into the equation of state the values:

$$p = p_0 - p_2, \qquad T = T_0 - T_1 - T_2, \qquad \rho = \rho_0 - \rho_2, \quad (48)$$
$$\rho_2 = \rho_0 x'^{-\frac{1}{2}} \Gamma_{v2},$$

and keep only the terms of the order $x'^{-\frac{1}{2}}$. We obtain:

$$p_2 + 2^{\frac{1}{2}} c x'^{\frac{1}{2}} \partial_{y'} p_2 + R\rho_2 T_1 - 2Rc^2 x' \partial_{y'} \rho_2 \partial_{y'} T_1 = 0. \quad (49)$$

Changing to z gives the expression:

$$C_{10} z^{\frac{1}{2}} \Gamma_{v2}' + C_{11}(z-1)\Gamma_{v2} + C_{12} u_0^{-2} \rho_0^{-1} p_0 \Pi_{v2} + C_{12}' u_0^{-2} \rho_0^{-1} p_0 \Pi_{v2}' = 0. \quad (50)$$

The solution around $z = 0$ is given by:

$$\Gamma_{v2} = N_0' + N_i' \log z + N_{2/3}' z^{2/3} + N_1' z + N_{4/3}' z^{4/3} + N_{5/3}' z^{5/3} + N_2' z^2 + N_{7/3}' z^{7/3} + N_{8/3}' z^{8/3} + N_3' z^3 + \cdots \quad (51)$$

As was done before, in Γ_{v2}' the derivative of the term $N_i' \log z$ is taken in the

form $N_i' z^{-i}$, and in Γ_{v2} this term itself is represented by an expansion according to the formula

$$\log z = (z-1) - \tfrac{1}{2}(z-1)^2 + (\tfrac{1}{3})(z-1)^3 - (\tfrac{1}{4})(z-1)^4 + \cdots, \qquad 2 > z > 0. \quad (52)$$

The symbol n denotes the highest power taken into account in the series expansion (52).

The series B_2, B_3, . . . come out from the expansion (52). One can represent:[10]

$$\log z = -(\text{Harm}) + nz - B_2 z^2 + B_3 z^3 - B_4 z^4 + \cdots. \quad (52a)$$

The number of terms in B_2, B_3, B_4, . . . etc. will depend on the number of terms taken into account in the series expansion (52).

B.C. is not fulfilled. For $z = 0$, the function $\Gamma_{v2} \to (-\infty)$ i.e., has a logarithmic singularity. The value of Γ_{v2} for $z = 0$ must be determined directly from the equation of state.

The solution around $z = 1$ is of the form:

$$\bar{\Gamma}_{v2} = P_0' + P_1' r + P_2' r^2 + P_3' r^3 + P_4' r^4 + \cdots. \quad (53)$$

P_0' must be determined from the boundary condition that $\bar{\Gamma}_{v2}(\xi) = 0$ for $\xi = \xi_{s2}$. The last B.C. is:

$$\begin{aligned} \rho_2'(y_{s2}') &= (3A)^{-\frac{1}{3}} \rho_0 x'^{-\frac{1}{3}} \bar{\Gamma}_2'(\xi_{s2}), \\ \bar{\Gamma}_2'(\xi_{s2}) &= \bar{\Gamma}_2'(1) + \cdots = 1.5 P_1' + \cdots = \text{const.}, \\ \bar{\Gamma}_2'(1) &\neq 0, \qquad \text{order:} \quad x'^{-\frac{1}{3}}. \end{aligned}$$

IV. THIRD APPROXIMATION.

1. *The Longitudinal Velocity.*

Put into the first equation of motion the values:

$$\begin{aligned} u &= u_0 - u_1 - u_2 - u_3, \qquad v = v_{v1} + v_{v2}, \qquad p = p_0 - p_2, \\ \rho &= \rho_0 - \rho_2, \qquad l = ac(x'^{\frac{1}{3}} + c_1 + c_2 x'^{-\frac{1}{3}}), \qquad u_3 \sim x'^{-\frac{1}{3}}, \end{aligned} \quad (54)$$

and keep only the terms of the order $x'^{-5/3}$. After all the necessary transformations we obtain:

$$\begin{aligned} &\rho_0(u_1 \partial_{x'} u_2 + u_2 \partial_{y'} u_1 - u_0 \partial_{x'} u_3 - v_{v1} \partial_{y'} u_2 - v_{v2} \partial_{y'} u_1) \\ &\quad + u_0 \rho_2 \partial_{x'} u_1 - c^2(4\rho_0 \partial_{y'} u_1 \partial_{y,} u_2 + u_0 \partial_{y'} u_1 \partial_{y'} \rho_2) - \partial_{x'} p_2 \\ &\quad - x'^{-\frac{1}{3}} 2c^2 c_1 \rho_0 (\partial_{y'} u_1)^2 - x'^{\frac{1}{3}} 4c^3 c_1 [\rho_0 \partial_{y'} u_1 (3\partial^2_{x',y'} u_1 + \partial^2_{y',y'} u_2)] \\ &\quad - x' 2c^2 \{2\rho_0 (\partial^2_{x',y'} u_1 \partial_{y'} u_2 + \partial_{y'} u_1 \partial^2_{x',y'} u_2) + u_0 \partial^2_{x',y'} u_1 \partial_{y'} \rho_2 \\ &\quad + \rho_0 [\partial_{y'} u_1 (\partial^2_{x',y'} u_2 + \partial^2_{y',y'} u_3 + \partial^2_{x',y'} v_{v1}) + \partial_{y'} u_2 (\partial^2_{x',y'} u_1 + \partial^2_{y',y'} u_2) \\ &\quad + \partial^2_{y',y'} u_1 \partial_{y'} u_3] - \partial_{y'} u_1 (\rho_2 \partial^2_{y',y'} u_1 + \partial_{y'} u_1 \partial_{y'} \rho_2)\} \\ &\qquad + x'^{\frac{1}{3}} 2c^3 \partial_{y'} u_1 \partial^2_{y',y'} u_1 \partial_{y'} \rho_2 = 0. \end{aligned} \quad (55)$$

An inspection shows significant differences between (55) and (144). Because the procedure of solving Eq. (55) remains the same as before, we shall not proceed further.

[10] The symbol "Harm" denotes a harmonic series which is a divergent series. But in practical applications only a finite number of terms will be used.

A comparison of the results of the present and the previous papers leads to the conclusions cited in the introduction.

APPENDIX—PART I.

I. $\pi_{xx} = \bar{p} - (\frac{4}{3})(\bar{a} + a') + (\frac{2}{3})(\bar{b} + b' + \bar{c} + c') + \bar{p}A + 2\bar{u}M_x + H_{xx}$,
$\pi_{yy} = \bar{p} - (\frac{4}{3})(\bar{b} + b') + (\frac{2}{3})(\bar{a} + a' + \bar{c} + c') + \bar{p}B + 2\bar{v}M_y + H_{yy}$,
$\pi_{zz} = \bar{p} - (\frac{4}{3})(\bar{c} + c') + (\frac{2}{3})(\bar{a} + a' + \bar{b} + b') + \bar{p}C + 3\bar{w}M_z + H_{zz}$,
$\pi_{yz} = \pi_{zy} = -2(\bar{h} + h') + \bar{p}H + \bar{u}M_y + \bar{v}M_x + H_{xy}$,
$\pi_{zx} = \pi_{xz} = -2(\bar{g} + g') + \bar{p}G + \bar{u}M_z + \bar{w}M_x + H_{zx}$,
$\pi_{xy} = \pi_{yx} = -2(\bar{f} + f') + \bar{p}F + \bar{v}M_z + \bar{w}M_y + H_{yz}$,
$\bar{a} = \bar{\mu}\partial_x\bar{u}$, $\bar{b} = \bar{\mu}\partial_y\bar{v}$, $\bar{c} = \bar{\mu}\partial_z\bar{w}$, $a' = \overline{\mu'\partial_x u'}$, $b' = \overline{\mu'\partial_y v'}$, $c' = \overline{\mu'\partial_z w'}$,
$2\bar{f} = \bar{\mu}(\partial_y\bar{w} + \partial_z\bar{v})$, $2f' = \overline{\mu'\partial_y w'} + \overline{\mu'\partial_z v'}$, $2\bar{g} = \bar{\mu}(\partial_z\bar{u} + \partial_x\bar{w})$,
$2g' = \overline{\mu'\partial_z u'} + \overline{\mu'\partial_x w'}$, $2\bar{h} = \bar{\mu}(\partial_x\bar{v} + \partial_y\bar{u})$, $2h' = \overline{\mu'\partial_x v'} + \overline{\mu'\partial_y u'}$,
$A = \overline{u'u'}$, $B = \overline{v'v'}$, $C = \overline{w'w'}$, $F = \overline{v'w'}$, $G = \overline{w'u'}$, $H = \overline{u'v'}$,
$H_{xx} = \overline{\rho'u'^2}$, $H_{yy} = \overline{\rho'v'^2}$, $H_{zz} = \overline{\rho'w'^2}$, $H_{xy} = \overline{\rho'u'v'}$, $H_{xz} = \overline{\rho'u'w'}$,
$H_{yz} = \overline{\rho'v'w'}$, $\bar{K} = \partial_t(\bar{\rho}\bar{T}) + \partial_x(\bar{u}\bar{\rho}\bar{T}) + \partial_y(\bar{v}\bar{\rho}\bar{T}) + \partial_z(\bar{w}\bar{\rho}\bar{T})$,
$K' = \overline{\rho'\partial_t T'} + \overline{T'\partial_t\rho'} + \partial_x\overline{(u'\rho'T')} + \partial_y\overline{(v'\rho'T')} + \partial_z\overline{(w'\rho'T')}$,
$\bar{N} = \bar{p}(\partial_x\bar{u} + \partial_y\bar{v} + \partial_z\bar{w}) + \overline{p'\partial_x u'} + \overline{p'\partial_y v'} + \overline{p'\partial_z w'}$,
$L = \partial_x(\bar{u}\,\overline{\rho'T'} + \bar{\rho}\,\overline{u'T'} + \bar{T}\,\overline{u'\rho'})$
$+ \partial_y(\bar{v}\,\overline{\rho'T'} + \bar{\rho}\,\overline{v'T'} + \bar{T}\,\overline{v'\rho'}) + \partial_z(\bar{w}\,\overline{\rho'T'} + \bar{\rho}\,\overline{w'T'} + \bar{T}\,\overline{w'\rho'})$,
$\bar{P} = \bar{k}(\partial_{xx}^2\bar{T} + \partial_{yy}^2\bar{T} + \partial_{zz}^2\bar{T})$, $P' = \overline{k'\partial_{xx}^2 T'} + \overline{k'\partial_{yy}^2 T'} + \overline{k'\partial_{zz}^2 T'}$,
$\bar{R} = (\frac{4}{3})\bar{\mu}[(\partial_x\bar{u})^2 + (\partial_y\bar{v})^2 + (\partial_z\bar{w})^2 - \partial_x\bar{u}\partial_y\bar{v} - \partial_x\bar{u}\partial_z\bar{w} - \partial_y\bar{v}\partial_z\bar{w}]$
$+ \bar{\mu}[(\partial_y\bar{u})^2 + (\partial_x\bar{v})^2 + (\partial_z\bar{w})^2 + (\partial_z\bar{u})^2 + (\partial_x\bar{v})^2 + (\partial_y\bar{w})^2] + 2\bar{\mu}[\partial_y\bar{w}\partial_z\bar{v}$
$+ \partial_x\bar{u}\partial_z\bar{w} + \partial_x\bar{v}\partial_y\bar{u}]$, $R' = (\frac{4}{3})[\overline{\mu'(\partial_x u')^2} + \overline{\mu'(\partial_y v')^2} + \overline{\mu'(\partial_z w')^2} - \overline{\mu'\partial_x u'\partial_y v'}$
$- \overline{\mu'\partial_x u'\partial_z w'} - \overline{\mu'\partial_y v'\partial_z w'}] + [\overline{\mu'(\partial_y u')^2} + \overline{\mu'(\partial_x v')^2} + \overline{\mu'(\partial_z w')^2} + \overline{\mu'(\partial_z u')^2}$
$+ \overline{\mu'(\partial_z v')^2} + \overline{\mu'(\partial_y w')^2}] + 2[\overline{\mu'\partial_y w'\partial_z v'} + \overline{\mu'\partial_z u'\partial_x w'} + \overline{\mu'\partial_x v'\partial_y u'}]$,
$S = (\frac{4}{3})\bar{\mu}[\overline{(\partial_x u')^2} + \overline{(\partial_y v')^2} + \overline{(\partial_z w')^2} - \partial_x\overline{u'\partial_y v'} - \partial_x\overline{u'\partial_z w'} - \partial_y\overline{v'\partial_z w'}] +$
$2\bar{\mu}[\partial_y\overline{w'\partial_z v'} + \partial_z\overline{u'\partial_x w'} + \partial_x\overline{v'\partial_y u'}] + \bar{\mu}[\overline{(\partial_y u')^2} + \overline{(\partial_z u')^2} + \overline{(\partial_x v')^2}$
$+ \overline{(\partial_z v')^2} + \overline{(\partial_x w')^2} + \overline{(\partial_y w')^2}] + (\frac{4}{3})[\partial_x\bar{u}(2\overline{\mu'\partial_x u'} - \overline{\mu'\partial_y v'} - \overline{\mu'\partial_z w'})$
$+ \partial_y\bar{v}(2\overline{\mu'\partial_y v'} - \overline{\mu'\partial_z u'} - \overline{\mu'\partial_z w'}) + \partial_z\bar{w}(2\overline{\mu'\partial_z w'} - \overline{\mu'\partial_z u'} - \overline{\mu'\partial_y v'})]$
$+ 2[\partial_y\bar{u}(\overline{\mu'\partial_y u'} + \overline{\mu'\partial_z v'}) + \partial_z\bar{u}(\overline{\mu'\partial_z u'} + \overline{\mu'\partial_x w'}) + \partial_x\bar{v}(\overline{\mu'\partial_x v'} + \overline{\mu'\partial_y u'})$
$+ \partial_z\bar{v}(\overline{\mu'\partial_z v'} + \overline{\mu'\partial_y w'}) + \partial_y\bar{w}(\overline{\mu'\partial_y w'} + \overline{\mu'\partial_z v'}) + \partial_z\bar{w}(\overline{\mu'\partial_z w'} + \overline{\mu'\partial_z u'})]$.

VI. 1. $a_3 = (-2)a_0$, $a_4 = (-a_1)$, $a_5 = (-\frac{8}{5})\beta_1$, $a_6 = (\frac{2}{5})a_0$, $a_7 = 0$,
$a_8 = (\frac{11}{70})\beta_1$, $a_9 = 0.1a_6$, etc., $\beta_1 = (\frac{1}{4})A_1^3(3A)^{\frac{1}{2}}$; $c_2 = 2\beta_1$, $c_3 = (\frac{1}{10})\beta_1$,
$c_4 = (-\frac{3}{81})\beta_1$, $c_5 = 5\beta_1$, etc. $C(g_4) = 0.4(0.5)^{5/2}a_3 + (\frac{2}{3})(0.5)^{7/2}a_5$
$+ 0.25(0.5)^4a_6 + 0.5(c_2 - c_3 + \cdots) + 0.25(1 - 0.5^4)(c_2 - 3c_3 + \cdots)$
$- 0.4(1 - 0.5^{5/2})(2c_2 - 3c_3 + \cdots) + (\frac{3}{11})(1 - 0.5^{11/2})(c_3 + \cdots) + \cdots$.

VI. 3. $G_2 = (\frac{1}{2})(3A)^{\frac{1}{3}}A_1{}^4[(-\frac{0.75}{4})z^{14/3} + (\frac{3.48}{11})z^{11/3} - 0.945z^{8/3}$
$+ 0.24z^{5/3} + 0.6z^{\frac{1}{3}}]$, $G_3 = (\frac{1}{2})(3A)^{\frac{1}{3}}A_1{}^2c_1(-3z^{8/3} + 4z^{5/3} - z^{\frac{1}{3}})$,
$G_{41} = (\frac{1}{2})(3A)^{\frac{1}{3}}[(\frac{3}{2})a_0z^{\frac{1}{3}} + 0.12a_3z^{5/3} + (\frac{3}{2})a_5z^{7/3} + (\frac{3}{2})a_6z^{8/3} + \cdots]$,
$G_{42} = (\frac{1}{2})(3A)^{\frac{1}{3}}\{c_2(z-1)^2 + [(\frac{7}{6})c_2 + c_3](z-1)^3$
$+ [(-\frac{5}{36})c_2 + (\frac{3}{4})c_3 + c_4](z-1)^4 + [(\frac{3.5}{10.5})c_2 - (\frac{3}{15})c_3$
$+ (\frac{11}{15})c_4 + c_5](z-1)^5 + \cdots\}$, $G_2(\xi_{e2}) = (\frac{1}{2})(3A)^{\frac{1}{3}}A_1{}^4(\frac{12.15}{77}) + \cdots$,
$G_2(1) \neq 0$, $G_3(\xi_{e2}) = (3A)^{\frac{1}{3}}A_1{}^2c_1Kx'^{-\frac{1}{3}} + \cdots$, $G_3(1) = 0$, $G_3'(1) \neq 0$,
$G_{42}(\xi_{e2}) = (\frac{8}{5})(3A)^{\frac{1}{3}}c_2K^2x'^{-1} + \cdots$, $G_{42}(1) = G_{42}'(1) = 0$, $G_{42}''(1) \neq 0$,
$\bar{G}_2 = A_1{}^4(-0.25\xi^6 + 1.16\xi^{9/2} - 2.52\xi^3 + 0.4\xi^{\frac{1}{2}} + 0.4) = (-0.81)A_1{}^4 + \cdots$,
$\bar{G}_2(1) \neq 0$, $\bar{G}_3 = A_1{}^2c_1[(-8)\xi^3 + (\frac{3.0}{3.5})\xi^{\frac{1}{2}} - (\frac{3}{5})] = (-2)A_1{}^2c_1 + \cdots$,
$\bar{G}_3(1) \neq 0$, $\bar{G}_{42} = 2c^2\xi^{\frac{1}{3}}(\xi^{\frac{1}{3}} - 1) + \cdots = 3c^2Kx'^{-\frac{1}{3}} + \cdots$,
$\bar{G}_{42}(1) = 0$, $\bar{G}_{42}'(1) \neq 0$.

VI. 4. $m_1 = (-2)(2m_0 - C'A_1{}^4)$, $m_2 = -(1.5m_1 + 1.6C'A_1{}^4)$,
$m_3 = [-(\frac{3}{5})m_2 + C'A_1{}^4]$, $m_4 = [-(\frac{1}{4})m_3 + (\frac{4}{11})C'A_1{}^4]$ $m_5 = (\frac{9}{35})C'A_1{}^4$,
$m_6 = (\frac{3}{11.35})C'A_1{}^4$, etc., $n_5 = (-\frac{4}{15})C'A_1{}^4$, $n_6 = (-\frac{1.7}{3.0})n_5$,
$n_7 = (-\frac{4.9}{5.9})n_6$, etc., $\bar{p}_1 = (\frac{1}{2})(\delta_2 - 8\bar{p}_0)$, $\bar{p}_2 = 0.1\delta_2 - (\frac{3}{4})\bar{p}_1$,
$\bar{p}_3 = (\frac{1}{3.9})(\delta_2 - 16\bar{p}_2)$, $\bar{p}_4 = (-\frac{1}{4})\bar{p}_3$, $\delta_2 = 8C'A_1{}^2c_1$, $\bar{p}_0 = -(\bar{p}_1\xi_{e2}^{3/2}$
$+ \bar{p}_2\xi_{e2}^3 + \bar{p}_3\xi_{e2}^{9/2} + \bar{p}_4\xi_{e2}^6)$, $\theta_{22}'(\xi_{e2}) = \theta_{22}'(1) + \cdots = (\frac{3}{2})\bar{p}_1 + 3\bar{p}_2 + \cdots$
$=$ const., $s_1 = (-4)s_0$, $s_2 = (-\frac{1}{3.0})\delta_2 - (\frac{3}{4})s_1$, $s_3 = (\frac{1}{3.9})(\delta_2 - 16s_2)$,
$s_4 = (-0.25)s_3$, $\delta_2 = 24C'A_1{}^2c_1$, $s_0 = \delta_3[(\frac{1}{3.0})\xi_{e2}^3 - (\frac{3}{8.5})\xi_{e2}^{9/2}$
$+ (\frac{3}{8.0.0})\xi_{e2}^6][1 - 4\xi_{e2}^{3/2} + 3\xi_{e2}^3 - (\frac{1.8}{1.3})\xi_{e2}^{9/2} + (\frac{4.5}{1.3})\xi_{e2}^6]^{-1}$, $\theta_{23}'(\xi)\,|_{\xi=\xi_{e2}}$
$= \theta_{23}'(1) + \cdots = (\frac{3}{2})s_1 + 3s_2 + \cdots =$ const., $E_3 = (-6)E_0$,
$E_5 = (-0.05)\delta_4$, $E_6 = 9E_0$, $E_8 = (\frac{1.0.9}{11.3.0})\delta_4$, etc., $\delta_4 = 18A_1{}^2(3A)^{\frac{1}{3}}C'$,
$F_0 = (-\frac{3.7}{10.0})\bar{\delta}_4$, $F_1 = (\frac{9}{8.0})\bar{\delta}_4$, $F_2 = (-\frac{1.8}{4.0})\bar{\delta}_4$, $F_3 = (-\frac{1}{3})\bar{\delta}_4$,
$F_5 = (-\frac{1.4}{13.15})\bar{\delta}_4$, etc., $\bar{\delta}_4 = 6(3A)^{\frac{1}{3}}A_1{}^2C'$, $F_4 = -(F_0t^{-4} + F_1t^{-3}$,
$+ F_2t^{-2} + F_3t^{-1} + F_4t + F_5t^2 + \cdots)$, $t = t(x')$, $\bar{\theta}_{24}'(\xi_{e3}) = \bar{\theta}_{24}'(1) + \cdots$
$= 1.5F_1 + \cdots$, $\bar{\theta}_{24}'(1) \neq 0$, order: $x'^{-\frac{1}{3}}$ (see eq. 112), $N_1 = [(-4)N_0$
$- (\frac{1}{2})M_0]$, $N_2 = 6N_0 - (\frac{1}{2})M_0 + 0.1M_1$, $N_{5/3} = (\frac{3}{35})M_{2/3}$,
$N_{8/3} = (\frac{1}{3.6})(3M_{5/3} - 49N_{5/3})$, $N_3 = (-4)N_0 + (\frac{1}{2})M_0 - (\frac{0.4}{3})M_1$
$+ (\frac{1}{3.6})M_2$, etc., $M_0 = (-2)C'[0.2A_1{}^4 + (\frac{1.0}{3.0})A_1{}^2c_1 + a_3]$,
$M_{2/3} = (-\frac{3.0}{5.0})C'a_5$, $M_1 = C'[9.4A_1{}^4 + (\frac{1.3.0}{3.0})A_1{}^2c_1 + 5a_3 - 2a_6]$,
$M_{5/3} = C'[5a_5 - (\frac{4.0}{3})a_8]$, $M_2 = C'[-20.4A_1{}^4 - (\frac{1.1.0}{3.0})A_1{}^2c_1 + 16a_6$
$- 24a_9]$, etc., $L_0 = (\frac{1}{3})[K_0 - (\frac{9}{15})K_1 + (\frac{9}{3.0})K_2 - (\frac{8.1.0}{3.3.0})K_3]$,
$L_1 = (\frac{1}{15})K_1 - (\frac{1}{3.0})K_2 + (\frac{8}{3.3.0})K_3$, $L_2 = (\frac{1}{10})[K_2 - (\frac{9}{11})K_3]$,
$L_3 = (\frac{1}{11})K_3$, $L_5 = (-\frac{1}{15})K_4$, etc., $K_0 = (-6)C'(0.81A_1{}^4 + 2A_1{}^2c_1)$,
$K_1 = (-4)C'(2.43A_1{}^4 + 12A_1{}^2c_1 - 3c_2)$, $K_2 = C'[-2.7A_1{}^4 - (\frac{1.3.0}{3.0})A_1{}^2c_1$
$+ 16c_2 + 18c_3]$, $K_3 = 3C'(0.32A_1{}^4 + 8c_3)$, etc., $L_4 = -(L_0t^{-4} + L_1t^{-3}$
$+ L_2t^{-2} + L_3t^{-1} + L_4t + L_5t^2 + \cdots)$, $t = t(x')$, $\bar{\theta}_{25}'(\xi_{e3}) = \bar{\theta}_{25}'(1) + \cdots$
$= 1.5L_1 + \cdots =$ const. $+ \cdots$, order: $x'^{-\frac{1}{3}}$ (see eq. 112).

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VI. 5. $L_2' = (3A)^{\frac{1}{2}}[(\frac{0.1}{3})A_1^4 + A_1^2c_1 + (\frac{1}{6})a_3]$, $L_4' = (\frac{1}{6})(3A)^{4/3}A_1 + 3(3A)^{-\frac{1}{2}}A_1^4c^2 + (\frac{5}{6})(3A)^{\frac{1}{2}}a_5 + 14A_1^2c_1]$, $L_7' = -[(\frac{1}{2})(3A)^{4/3}A_1 + 1.5(3A)^{-\frac{1}{2}}A_1^4c^2 + (\frac{5}{6})(3A)^{\frac{1}{2}}a_5 + (\frac{5}{6})(3A)^{\frac{1}{2}}a_8]$, $L_8' = (3A)^{\frac{1}{2}}[0.945A_1^4 + (\frac{3.2}{6})A_1^2c_1 - (\frac{1}{3})a_6 - (\frac{1}{3})a_9]$, $L_{10} = (-2)[(-\frac{1}{6})(3A)^{4/3}A_1 + 3(3A)^{-\frac{1}{2}}A_1^4c^2 + 2(3A)^{\frac{1}{2}}a_8]$, $L_{11}' = -(3A)^{\frac{1}{2}}(1.15A_1^4 + 5a_9)$, $L_{13}' = (\frac{5}{6})(3A)^{-\frac{1}{2}}A_1^4c^2$, $L_{14}' = 0.3(3A)^{\frac{1}{2}}A_1^4$, etc., $M_1' = (-\frac{1}{2})(3A)^{\frac{1}{2}}A_1^2(0.81A_1^2 + 2c_1)$, $M_2' = (3A)^{\frac{1}{2}}[(\frac{1}{6})(3A)^{\frac{1}{2}}A_1 - (\frac{1.3}{3})A_1^2c_1 - 0.945A_1^4 + c_2 + c_3]$, $M_3' = (3A)^{\frac{1}{2}}[(\frac{7}{1.5})(3A)^{\frac{1}{2}}A_1 - 0.135A_1^4 + (\frac{1.0}{6})A_1^2c_1 + (\frac{1}{3})c_2 - (\frac{1}{3})c_3] + 7.5(3A)^{-\frac{1}{2}}A_1^4c^2$, $M_4' = 7(3A)^{-\frac{1}{2}}A_1^4c^2 - (3A)^{\frac{1}{2}}[0.025A_1^4 + (\frac{1.4}{3.7})A_1^2c_1 + (\frac{1}{3})c_2 - 3.5c_3 - 6c_4] + (\frac{5}{3.4})(3A)^{4/3}A_1$, $M_5' = (\frac{7}{3})(3A)^{-\frac{1}{2}}A_1^4c^2 - (3A)^{\frac{1}{2}}[(\frac{5.95}{18})A_1^4 - (\frac{1.1.2}{3.1})A_1^2c_1 - (\frac{3}{3.7})c_2 - (\frac{5}{6})c_3 - 4c_4] + (\frac{1.6}{1.8.8})(3A)^{4/3}A_1$, etc., $M_0' = -[M_1't + (\frac{1}{2})M_2't^2 + (\frac{1}{3})M_3't^3 + \cdots]$, $p_2'(y'_{s2}) = (3A)^{-\frac{1}{2}}u_0^2\rho_0p_0^{-1}x'^{-2}\overline{\Pi}_2'(\xi_{s2}) = (\frac{3}{2})(3A)^{-\frac{1}{2}}u_0^2\rho_0p_0^{-1}x'^{-2}M_1' + \cdots$, $\overline{\Pi}_2'(1) \neq 0$, order: x'^{-2}.

VI. 6. $N_0' = (-C_{12}/C_{11})L'$, $N_1' = (\frac{3}{4})(C_{12}/C_{10})L_2'$, $N_{4/3}' = (\frac{3}{4})(C_{12}/C_{10})[(\frac{3}{4})(C_{11}/C_{10})L_2' + 2L' + L_3']$, $N_{5/3}' = (\frac{3}{5})C_{10}^{-1}[C_{11}N_{4/3}' + (\frac{3}{4})C_{12}L_4']$, $N_2' = (\frac{1}{2})C_{10}^{-1}[C_{10}N_1' + C_{11}N_{5/3}' + (\frac{3}{5})L_5']$, etc., $C_{10} = (-18)(3A)^{-4/3}A_1^2C'c^2$, $C_{11} = 4A_1^2C'$, $C_{12} = 4u_0^2\rho_0p_0^{-1}$, $P_1'' = (-\frac{1}{2})(C_{12}/C_{10})M_0'$, $P_1' = (C_{12}/C_{10})[(\frac{3}{4})M_0' - M_1']$, $P_2' = (-\frac{1}{3})P_1' + (\frac{1}{6})P_1'' - (\frac{1}{2})(C_{11}/C_{10})[P_0' - (\text{Harm})P_1'']$ $- (\frac{1}{4})(C_{12}/C_{10})M_2'$, $P_3' = (\frac{1}{6})\{(-4)P_2' + [(\frac{1}{3}) - 3(C_{11}/C_{10})]P_1'$ $- (C_{12}/C_{10})M_3'\}$ etc., $P_0' = -(P_1''\log t^2 + P_1't + P_2't^2 + \cdots)$.

VI. 7. $S_1' = (\frac{3}{4})A_1[(3A)^{\frac{1}{2}}A - (9A_1 + 8)(3A)^{-\frac{1}{2}}A_1^2c^2]u_0^2\rho_0p_0^{-1}$, $S_2' = (-\frac{3}{4})A_1[2.5(3A)^{\frac{1}{2}}A - (33A_1 - 48)(3A)^{-\frac{1}{2}}A_1^2c^2]u_0^2\rho_0p_0^{-1}$, $S_3' = (\frac{3}{4})A_1[0.5(3A)^{\frac{1}{2}}A - (15A_1 - 24)(3A)^{-\frac{1}{2}}A_1^2c^2]u_0^2\rho_0p_0^{-1}$, $S_4' = (-3)(3A)^{-\frac{1}{2}}A_1^2c^2u_0^2\rho_0p_0^{-1}$, $S_5' = -(S_1'f^3 + S_2'ff'^2 + S_3'f^2f' + S_4'f'^3)$, $f = f(x') = \xi_{s2} = (1 + Kx'^{-\frac{1}{2}})$, $p_{2c}'(y'_{s2}) = p_0(3A)^{-\frac{1}{2}}x'^{-2}\Pi_{2c}'(\xi)\,|_{\xi=\xi_{s2}}$, $\Pi_{2c}'(\xi_{s2}) = 4.5S_4' + \cdots$, $\Pi_{2c}'(1) \neq 0$.

VII. 1. $L(H_1') + (\frac{1}{3})A_1^{-2}[(2\bar{c}_2 + c_1^2) + 4c_1(3A)^{\frac{1}{2}}z^{\frac{1}{2}}](3zf'' + f')f' = 0$, $L(H_2) + (\frac{3}{2})[f + (\frac{1}{2})zf']\Sigma g = 0$, $L(H_3) - (\frac{1}{3})A_1^{-2}[(\frac{1}{3})(3A)^{\frac{1}{2}}z(8f' + 3z^{\frac{1}{2}}f'') - 2c_1(3zf'' + 2f')]\Sigma g' = 0$, $L(H_4) - (\frac{3}{4})A_1^{-2}[(\frac{1}{4})(3A)^{\frac{1}{2}}z^{\frac{1}{2}} - c_1]\cdot zf'\Sigma g'' = 0$, $L(H_5) + (\frac{1}{3})A_1^{-2}(3z\Sigma g'' + \Sigma g')\Sigma g' = 0$, $L(H_6) - 1.5(3A)^{-\frac{1}{2}}z^{\frac{1}{2}}f'G = 0$, $L(H_7) + (\frac{1}{2})[f + (\frac{3}{4})zf'(2 - A_1^{-2}f'')] - (\frac{1}{4})A_1^{-2}f'^2]\Gamma_2 = 0$, $L(H_8) - (\frac{1}{3})A_1^{-2}[(\frac{1}{3})(3A)^{\frac{1}{2}}z^{\frac{1}{2}}(8f' + 3zf'') - [c_1 + (\frac{1}{2})f](3zf'' + 2f') + (\frac{8}{3})c^2(3A)^{-\frac{1}{2}}z^{\frac{1}{2}}f'[zf'' + (\frac{1}{2})f']\Gamma_2' + (\frac{8}{3})A_1^{-2}\{[c_1 - (\frac{1}{2})(2c^2)^{\frac{1}{2}}z^{\frac{1}{2}} + (\frac{1}{4})f]zf' - (\frac{3}{4})c^2(3A)^{-\frac{1}{2}}z^{4/3}f'^2\}\Gamma_2'' = 0$, $L(H_9) + (\frac{1}{3})A_1^{-2}[3z(\Gamma_2'\Sigma g'' + \Gamma_2''\Sigma g') + 2\Gamma_2'\Sigma g'] = 0$, $L(H_{10}) - 1.5u_0^{-2}p_0\rho_0[\Pi_2 + (\frac{1}{3})z\Pi_2'] = 0$, $L(H_i) = (\frac{3}{4})z(z-1)H_i'' + (\frac{1}{2})(z-1)H_i' - (\frac{3}{8})H_i$.

(H_1). $H_1 = \bar{a}_0 + \bar{a}_1 z + \bar{a}_{5/3} z^{5/3} + \bar{a}_2 z^2 + \bar{a}_{8/3} z^{8/3} + \bar{a}_3 z^3 + \cdots$,
$\bar{a}_1 = (-3)\bar{a}_0 + 0.5C_{14}$, $\bar{a}_{5/3} = (\frac{3}{20})C_{15}$, $\bar{a}_2 = 1.2\bar{a}_0 - 0.7C_{14}$,
$\bar{a}_{8/3} = (\frac{1}{28})(\bar{a}_{5/3} - 7.5C_{15})$, $\bar{a}_3 = (\frac{1}{6})(\bar{a}_2 + C_{14})$, etc.,
$C_{14} = 2A_1^2(2\bar{c}_2 + c_1^2)$, $C_{15} = 16(3A)^{\frac{1}{2}}c_1$. B.C. $H_1'(y') = H_1'(\xi) = 0$
for $y' = \xi = 0$. $\bar{H}_1 = \bar{d}_1(z-1) + \bar{d}_2(z-1)^2 + \bar{d}_3(z-1)^3 + \cdots$,
$\bar{d}_2 = (\frac{3}{4})\bar{d}_1 - (\frac{1}{4})(C_{14} + C_{15})$, $\bar{d}_3 = (-\frac{1}{6})(2\bar{d}_2 + 2C_{14} + 3C_{15})$,
$\bar{d}_4 = -(\frac{1}{4})\bar{d}_3 + \cdots$. B.C. $\bar{H}_1(\xi_{s2}) = 0$ if $\bar{d}_1 = -(\bar{d}_2 l + \bar{d}_3 l^2 + \bar{d}_4 l^3 + \cdots)$.
$\bar{H}_1'(\xi_{s2}) = \bar{H}_1'(1) + \cdots = (\frac{3}{2})\bar{d}_1 + \cdots$, $\bar{H}_1'(1) \neq 0$, $\bar{d}_1 \sim x'^{-\frac{1}{2}}$,
order: $x'^{-5/2}$ (with 151).

(H_2). $H_2 = \bar{f}_0 + \bar{f}_1 z + \bar{f}_2 z^2 + \bar{f}_{8/3} z^{8/3} + \bar{f}_3 z^3 + \cdots$, $\bar{f}_1 = (\frac{1}{6})\bar{e}_0 + 3\bar{f}_0$,
$\bar{f}_2 = -(0.4\bar{e}_0 + 0.1\bar{e}_1 + 2.4\bar{f}_0)$, $\bar{f}_{8/3} = (\frac{3}{28})\bar{e}_{5/3}$, $\bar{f}_3 = (\frac{1}{6})[8\bar{f}_2 + (\frac{1}{6})\bar{e}_2]$,
etc., $\bar{e}_0 = 6A_1^2[a_0 + (0.4A_1^2 - (\frac{2}{3})c_1)A_1^2]$, $\bar{e}_1 = 6A_1^2\{(-3)a_0 + a_3$
$- [A_1^2 - (\frac{1}{28})c_1]A_1^2\}$, $\bar{e}_{5/3} = 6A_1^2 a_5$, $\bar{e}_2 = 6A_1^2[2a_0 - 3a_3 + a_6$
$- (0.64A_1^2 + 14c_1)A_1^2]$, etc. $\bar{H}_2 = \bar{k}_3(z-1)^3 + \bar{k}_4(z-1)^4$
$+ \bar{k}_5(z-1)^5 + \cdots$, $\bar{k}_3 = (-\frac{1}{168})\bar{j}_2$, $\bar{k}_4 = (\frac{1}{28})(\bar{j}_2 - \bar{j}_3)$,
$\bar{k}_5 = [-(\frac{1}{1680})(\bar{j}_2 - \bar{j}_3) - \frac{1}{60}\bar{j}_4]$, etc., $\bar{j}_2 = 6A_1^4(-0.81A_1^2 + 2c_1)$,
$\bar{j}_3 = 6A_1^2[c_2 - (1.89A_1^2 + (\frac{20}{3})c_1)A_1^2]$, $\bar{j}_4 = 6A_1^2\{2c_2 + c_3$
$- [0.45A_1^2 + (\frac{10}{3})c_1]A_1^2\}$, $\bar{j}_5 = 6A_1^2(2c_3 + c_4 + 0.13A_1^4)$,
$\bar{j}_6 = 6A_1^2(2c_4 + c_5 - 0.1A_1^4)$, etc. B.C. $\bar{H}_2(\xi_{s2}) = (\frac{27}{8})\bar{k}_3 K^2 x'^{-\frac{1}{2}} + \cdots$,
$\bar{H}_2(\xi_{s2}) \neq 0$, order: x'^{-3}, $\bar{H}_2'(\xi_{s2}) = (\frac{81}{8})\bar{k}_3 K^2 x'^{-1} + \cdots$, $\bar{H}_2'(1)$
$= \bar{H}_2''(1) = 0$, $\bar{H}_2'''(1) \neq 0$, order: x'^{-3}.

(H_3). $H_3 = \delta_0' + \delta_1' z + \delta_{5/3}' z^{5/3} + \delta_2' z^2 + \delta_{8/3}' z^{8/3} + \delta_3' z^3 + \cdots$,
$\delta_1' = (-3)\delta_0' - 2\bar{b}_0$, $\delta_{5/3}' = (-\frac{3}{2})\bar{b}_{2/3}$, $\delta_2' = (-0.4)(\delta_1' + \bar{b}_1)$,
$\delta_{8/3}' = (\frac{1}{2})[(\frac{1}{6})\delta_{5/3}' - \bar{b}_{5/3}]$, $\delta_3' = (\frac{1}{3})(\delta_2' - \bar{b}_2)$, etc., $\bar{b}_0 = c_1[0.2A_1^4$
$+ (\frac{10}{3})A_1^2 c_1 + a_3]$, $\bar{b}_{2/3} = (\frac{3}{2})a_5 c_1$, $\bar{b}_1 = (-A_1^4)[0.2(3A)^{\frac{1}{2}} + 2.18c_1]$
$-(\frac{1}{3})A_1^2 c_1[10(3A)^{\frac{1}{2}} + 41c_1] - a_3[(3A)^{\frac{1}{2}} + (\frac{2}{3})c_1] + 2a_6 c_1$,
$\bar{b}_{5/3} = (\frac{3}{8})(3A)^{\frac{1}{2}}[0.2A_1^4 + (\frac{10}{3})A_1^2 c_1 + a_3] - (\frac{3}{2})a_5[(3A)^{\frac{1}{2}} + (\frac{2}{3})a_5]$
$+ (\frac{3}{8})a_8 c_1$, $\bar{b}_2 = (3A)^{\frac{1}{2}}[1.7A_1^4 + (\frac{28}{3})A_1^2 c_1 + a_3 - 2a_6]$
$+ c_1[5.07A_1^4 + (\frac{40}{3})A_1^2 c_1 - 5a_6 + 3a_9]$, $\bar{b}_{7/3} = (\frac{3}{2})(3A)^{\frac{1}{2}}a_5$ etc.
$\bar{H}_3 = \bar{m}_0 + \bar{m}_1(z-1) + \bar{m}_2(z-1)^2 + \cdots$, $\bar{m}_0 = (\frac{1}{6})\bar{l}_0$,
$\bar{m}_2 = (\frac{1}{3})[2\bar{m}_1 - (\frac{1}{4})\bar{l}_1]$, $\bar{m}_3 = (-\frac{1}{6})[2\bar{m}_2 + (\frac{1}{4})\bar{l}_2]$,
$\bar{m}_4 = (-\frac{1}{4})[\bar{m}_3 + \frac{1}{168}\bar{l}_3]$, etc., $\bar{l}_0 = (1.5[(3A)^{\frac{1}{2}} - 4c_1](-0.81A_1^4 + 2A_1^2 c_1)$,
$\bar{l}_1 = (\frac{1}{3})[(3A)^{\frac{1}{2}}(12.03A_1^4 + 42A_1^2 c_1 - 6c_2) - 4c_1(5.67A_1^4 + 26A_1^2 c_1$
$- 6c_2)]$, $\bar{l}_2 = (-\frac{1}{6})[(3A)^{\frac{1}{2}}(-18.63A_1^4 - 208A_1^2 c_1 + 78c_2 + 27c_3)$
$-A_1^4(42.12c_1 + 23.49) + 2A_1^2 c_1(160c_1 - 29) + 4c_1(30c_2 - 27c_3)]$,
$\bar{l}_3 = (-\frac{1}{6})[(3A)^{\frac{1}{2}}(12.33A_1^4 + 117c_3) - A_1^4(23.4c_1 + 15.66) - (\frac{484}{3})A_1^2 c_1$
$-18c_1 c_2 + 58c_2]$, etc., B.C. $\bar{H}_3(\xi_{s2}) = 0$ if $\bar{m}_1 = -(\bar{m}_0 l^{-1} + \bar{m}_2 l$
$+ \bar{m}_3 l^2 + \cdots)$, $\bar{H}_3'(\xi_{s2}) = 1.5\bar{m}_1 + \cdots$, $\bar{H}_3'(1) \neq 0$, $\bar{m}_1 \sim x'^{\frac{1}{2}}$,
order: $x'^{-\frac{1}{2}}$.

(H_4). $H_4 = \bar{F}_0 + \bar{F}_1 z + \bar{F}_{5/3} z^{5/3} + \bar{F}_2 z^2 + \bar{F}_{7/3} z^{7/3} + \bar{F}_{8/3} z^{8/3} + \cdots$,
$\bar{F}_1 = (-3)\bar{F}_0$, $\bar{F}_{5/3} = (\frac{5}{20})\bar{n}_{2/3}$, $\bar{F}_2 = (-0.4)\bar{F}_1 + 0.1\bar{n}_1$, $\bar{F}_{7/3} = (\frac{1}{14})\bar{n}_{4/3}$,
$\bar{F}_{8/3} = (\frac{1}{56})(3\bar{n}_{5/3} + 2\bar{F}_{5/3})$, $\bar{F}_3 = (\frac{1}{24})(4\bar{F}_2 + \bar{n}_2)$, etc., $\bar{n}_{2/3} = (-\frac{20}{3})a_5 c_1$,
$\bar{n}_1 = (-12)[(-0.84)A_1^4 - (\frac{8}{3})A_1^2 c_1 + a_6]c_1$, $\bar{n}_{4/3} = (\frac{8}{3})(3A)^{\frac{4}{3}}a_5$,
$\bar{n}_{5/3} = -\{[(\frac{20}{3})c_1(4a_5 + a_3)] - 3(3A)^{\frac{4}{3}}[(-0.84)A_1^4 - (\frac{8}{3})A_1^2 c_1 + a_6]\}$,
$\bar{n}_2 = -[A_1^4(23.2 + 10.08c_1) + 32A_1^2 c_1^2 + 12a_6 c_1 + 80a_9]$,
$\bar{n}_{7/3} = -(\frac{8}{3})(3A)^{\frac{4}{3}}(a_5 - 4a_8)$, etc. $\bar{H}_4 = \bar{K}_1(z-1)$
$+ \bar{K}_2(z-1)^2 + \bar{K}_3(z-1)^3 + \cdots$, $\bar{K}_2 = (\frac{1}{3})[2\bar{K}_1 - (\frac{1}{2})\bar{J}_1]$,
$\bar{K}_3 = (-\frac{1}{3})[2\bar{K}_2 + (\frac{1}{2})\bar{J}_2]$, $\bar{K}_4 = (-\frac{1}{4})[\bar{K}_3 + (\frac{1}{18})\bar{J}_3]$, etc.,
$J_1 = (-6)[(\frac{1}{3})(3A)^{\frac{1}{3}} - 2c_1][(-0.27A_1^4 - (\frac{8}{3})A_1^2 c_1 + c_2]$,
$J_2 = (-3)\{(3A)^{\frac{1}{3}}[(-0.18)A_1^4 - (\frac{4.0}{9})A_1^2 c_1 + (\frac{8}{3})c_2 + 3c_3]$
$- 4c_1[(-\frac{8}{3})A_1^2 c_1 + c_2 + 3c_3]$, $J_3 = (-3)\{(3A)^{\frac{1}{3}}[(-\frac{4.0}{3.7})A_1^2 c_1$
$+ (\frac{8}{3})c_2 + 5c_3 + 6c_4] - 12c_1(-0.01A_1^4 + c_3 + 2c_4)\}$, etc.
B.C. $\bar{H}_4(\xi_{*2}) = 0$ if $\bar{K}_1 = -(\bar{K}_2 t + \bar{K}_3 t^2 + \bar{K}_4 t^3 + \cdots)$,
$\bar{H}_4'(\xi_{*2}) = 1.5\bar{K}_1 + \cdots$, $\bar{H}_4'(1) \neq 0$, $\bar{K}_1 \sim x'^{-\frac{1}{3}} + \cdots$, order: $x'^{-5/2}$.

(H_5). $H_5 = \bar{B}_0 + \bar{B}_1 z + \bar{B}_2 z^2 + \cdots$, $\bar{B}_1 = (-3)\bar{B}_0 + (\frac{1}{3})\bar{A}_0$,
$\bar{B}_2 = (-0.4)\bar{B}_1 + 0.1\bar{A}_1$, $\bar{B}_3 = (\frac{1}{6})[\bar{B}_2 + (\frac{1}{4})\bar{A}_2]$, etc., $\bar{A}_0 = (\frac{1}{2})A_1^{-2}\varphi_1^2$,
$\bar{A}_{2/3} = (\frac{1.0}{3})A_1^{-2}\varphi_1$, $\bar{A}_1 = 4A_1^{-2}[-(0.84)A_1^4 - (\frac{8}{3})A_1^2 c_1 + a_6]\varphi_1$,
$\bar{A}_{4/3} = (\frac{2.0}{72})A_1^{-2}a_5^2$, $\bar{A}_{5/3} = (\frac{1}{3})A_1^{-2}\{(-35)[0.84A_1^4 + (\frac{8}{3})A_1^2 c_1 - a_6]a_5$
$+ 28a_8\varphi_1\}$, $\bar{A}_2 = A_1^{-2}\{4[0.84A_1^4 + (\frac{8}{3})A_1^2 c_1 - a_6]^2 + (2.61A_1^4 + 9a_9)\varphi_1\}$,
$\bar{A}_{7/3} = (\frac{3.0}{8})A_1^{-2}a_5^2 a_8^2$, $\bar{A}_{8/3} = (\frac{1}{3})A_1^{-2}\{(-36.1)A_1^4 - (\frac{1.8.0}{3})[(\frac{8}{3})A_1^2 c_1$
$+ a_6]a_5 + 30a_4 a_9\}$, etc., $\varphi_1 = [0.2A_1^4 + (\frac{1.0}{3})A_1^2 c_1 + a_3]$.
$\bar{H}_5 = \bar{E}_0 + \bar{E}_1(z-1) + \bar{E}_2(z-1)^2 + \cdots$, $\bar{E}_0 = (\frac{1}{3})\bar{D}_0$,
$\bar{E}_2 = (\frac{1}{3})[2\bar{E}_1 - (\frac{1}{2})\bar{D}_1]$, $\bar{E}_3 = (-\frac{1}{3})[2\bar{E}_2 + (\frac{1}{2})\bar{D}_2]$,
$\bar{E}_4 = (-\frac{1}{4})[\bar{E}_3 + (\frac{1}{18})\bar{D}_3]$, etc., $\bar{D}_0 = (\frac{1}{2})A_1^{-2}\{[(-1.35)A_1^4$
$- (\frac{3.2}{3})A_1^2 c_1 + 2c_2]\varphi_2\}$, $\bar{D}_1 = A_1^{-2}\{[-1.08A_1^4 - (\frac{1.4}{3})A_1^2 c_1 + c_2]\varphi_2$
$+ [(-\frac{8}{3})A_1^2 c_1 + c_2 + 3c_3]\varphi_2\}$, $\bar{D}_2 = A_1^{-2}\{[-0.135A_1^4 - (\frac{1.8}{3})A_1^2 c_1$
$+ 2c_2 + 4.5c_3]\varphi_3 + [0.51A_1^4 + 6(c_3 + c_4)]\varphi_2\}$, $\bar{D}_3 = A_1^{-2}\{[0.51A_1^4$
$+ 6(c_3 + c_4)]\varphi_3 + [(-\frac{8}{3})A_1^2 c_1 + c_2 + 3c_3](0.27A_1^4 + 3c_3)\}$, etc.,
$\varphi_2 = (-0.81A_1^2 - 2c_1)A_1^2$, $\varphi_3 = [-0.54A_1^4 - (\frac{1.8}{3})A_1^2 c_1 + 2c_2]$.
B.C. $\bar{H}_5(\xi_{*2}) = 0$ if $\bar{E}_1 = -(\bar{E}_0 t^{-1} + \bar{E}_2 t + \bar{E}_3 t^2 + \cdots)$,
$\bar{H}_5'(\xi_{*2}) = (\frac{3}{4})\bar{E}_1 + \cdots$, $\bar{H}_5'(1) \neq 0$, $\bar{E}_1 \sim x'^{\frac{1}{3}} + \cdots$, order: $x'^{-\frac{1}{3}}$.

(H_6). $H_6 = i_0 + i_1 z + i_2 z^2 + \cdots$, $i_1 = (\frac{1}{3})s_1 - 3i_0$, $i_2 = (-0.1)s_1 + 1.2i_0$,
$i_3 = (\frac{1}{6})i_2 + s_2$, $i_4 = (-\frac{1}{4})i_3 + s_3$, etc., $s_1 = (-6)A_1^3[(-0.6A_1^3 + c_1)A_1^2$
$-1.5a_0]$, $s_2 = (-6)A_1^3[(0.36A_1^3 - 5c_1)A_1^2 + 1.5a_0 - 1.2a_3]$,
$s_3 = (-6)A_1^3[(1.185A_1^2 + 7c_1)A_1^2 + 1.2a_3 - (\frac{8}{3})a_5]$, etc.
$\bar{H}_6 = w_1(z-1) + w_2(z-1)^2 + w_3(z-1)^3 + \cdots$, $w_2 = (\frac{1}{3})[2w_1 - (\frac{1}{2})\bar{u}_1]$,
$w_3 = (-\frac{1}{3})[w_2 + (\frac{1}{6})\bar{u}_2]$, $w_4 = (-\frac{1}{4})[w_3 + (\frac{1}{18})\bar{u}_3]$, etc.,

$\tilde{u}_1 = (-\frac{75.9}{77})A_1^6$, $\tilde{u}_2 = 12A_1^4[(\frac{58.32}{77})A_1^2 + c_1]$, $\tilde{u}_3 = (-6)A_1^2[(-\frac{13.54}{11})A_1^4 - 5A_1^2c_1 + c_2]$, $\tilde{u}_4 = 6A_1^2[(\frac{37.2}{11})A_1^4 + 3A_1^2c_1 - (\frac{8}{5})c_2 - c_3]$, etc.
B.C. $\bar{H}_6(\xi_{s2}) = 0$ if $\bar{w}_1 = -(\bar{w}_2t + \bar{w}_3t^2 + \cdots)$. $\bar{H}_6'(\xi_{s2}) = 1.5\bar{w}_1 + \cdots$, $\bar{H}_6'(1) \neq 0$, $\bar{w}_1 \sim x'^{-\frac{1}{2}} + \cdots$, order: $x'^{-5/2}$.

(H_8). $H_8 = \tilde{G}_0 + \tilde{G}_1z + \tilde{G}_{4/3}z^{4/3} + \tilde{G}_{5/3}z^{5/3} + \tilde{G}_2z^2 + \tilde{G}_{7/3}z^{7/3} + \cdots$,
$\tilde{G}_1 = (\frac{1}{2})\tilde{I}_0 - 3\tilde{G}_0$, $\tilde{G}_{4/3} = (\frac{1}{3})\tilde{I}_{1/3}$, $\tilde{G}_{5/3} = (\frac{3}{10})\tilde{I}_{2/3}$, $\tilde{G}_2 = 0.1\tilde{I}_1 - 0.4\tilde{G}_1$,
$\tilde{G}_{7/3} = (-\frac{3}{35})[3\tilde{G}_{4/3} + (\frac{1}{2})\tilde{I}_{4/3}]$, $\tilde{G}_{8/3} = (\frac{1}{8})[(\frac{1}{3})\tilde{G}_{5/3} + (\frac{1}{2})\tilde{I}_{5/3}]$, etc.,
$\tilde{I}_0 = (-N_1')(A_1^2 + 2c_1)$, $\tilde{I}_{1/3} = (-\frac{1}{4})N_1'(3A)^{4/3} - 2N_{4/3}'(A_1^2 + 2c_1)$,
$\tilde{I}_{2/3} = 4N_1^2(3A)^{\frac{1}{3}} - (\frac{2}{3})N_{4/3}'(3A)^{4/3}c - (\frac{10}{3})N_{5/3}'(A_1^2 + 2c_1)$,
$\tilde{I}_1 = (\frac{1}{2})N_1'(9A_1^2 + 10c_1) + 5N_{4/3}'(3A)^{\frac{1}{3}} - (\frac{5}{4})N_{5/3}'(3A)^{4/3}$
$- 5N_2'(A_1^2 + 2c_1)$, $\tilde{I}_{4/3} = N_1'(3A)^{4/3}c + 8N_{4/3}'(A_1^2 + c_1)$
$+ (\frac{8}{3})N_{5/3}'[10(3A)^{\frac{1}{3}} + (2c^2)^{\frac{1}{3}}] - 2N_2'(3A)^{4/3}c - 7N_{7/3}'(A_1^2 + 2c_1)$,
$\tilde{I}_{5/3} = (-5.5)N_1'(3A)^{4/3} + 6N_{4/3}'(3A)^{4/3}c + (\frac{8}{5})N_{5/3}'(15A_1^2 + 14c_1)$
$+ N_2'[3(2c^2)^{\frac{1}{3}} + 4(3A)^{\frac{1}{3}}] - (\frac{88}{15})N_{7/3}'(3A)^{4/3} - (\frac{28}{3})N_{8/3}'(A_1^2 + 2c_1)$,
$\tilde{I}_2 = (-6)N_1'A_1^2 - (\frac{3}{5})N_{4/3}'[11(3A)^{\frac{1}{3}} + (2c^2)^{\frac{1}{3}}] + (\frac{10}{3})N_{5/3}'(3A)^{4/3}c$
$+ 2N_2'(9A_1^2 + 8c_1) + (\frac{14}{3})N_{7/3}'[2(3A)^{\frac{1}{3}} + (2c^2)^{\frac{1}{3}}] - 4N_{8/3}'(3A)^{4/3}c$
$-9N_3'(A_1^2 + 2c_1)$, etc., $\bar{H}_8 = \bar{P}_0 + \bar{P}_1\log(z-1) + \bar{P}_1(z-1)$
$+ \bar{P}_2(z-1)^2 + \cdots$, $\bar{P}_1 = (\frac{1}{6})\bar{T}_{-1}$, $\bar{P}_2 = 2\bar{P}_1 - (\frac{1}{2})\bar{T}_1$,
$\bar{P}_3 = (-\frac{1}{3})[\bar{P}_2 - \alpha_2 + (\frac{1}{6})\bar{T}_2]$, $\alpha_0 = (-\frac{1}{3})[\bar{P}_1 - (\frac{1}{2})\bar{T}_0]$, etc., $\alpha_0 = \bar{P}_0$
$-$ (Harm) $\bar{P}_1$, $\alpha_2 = \bar{P}_2 + (2 + \cdots)\bar{P}_1$, $\alpha_4 = \bar{P}_4 - [(\frac{1}{3}) + 1 + \cdots]\bar{P}_1$,
etc., $\bar{T}_{-1} = 3[-(3A)^{\frac{1}{3}} - 4c_1 + (2c^2)^{\frac{1}{3}}]P_1''$, $\bar{T}_0 = -[1.5(3A)^{\frac{1}{3}} + 3c_1]P_1'$
$+ (\frac{1}{2})[18(3A)^{\frac{1}{3}} - (3A)^{4/3}c + 10(2c^2)^{\frac{1}{3}}]P_1''$, $\bar{T}_1 = (-3)[(3A)^{\frac{1}{3}}$
$+ (2c^2)^{\frac{1}{3}}]P_2' + (\frac{1}{2})[9(3A)^{\frac{1}{3}} + 10c_1 - (3A)^{4/3}c]P_1' - (\frac{1}{3})[13(3A)^{\frac{1}{3}}$
$+ 3.5(3A)^{4/3}c + 10c_1 - 5(2c^2)^{\frac{1}{3}}]P_1''$, etc. B.C. $\bar{H}_8(\xi_{s2}) = 0$
if $\bar{P}_1 = -(\bar{P}_0t^{-1} + \bar{P}_1t^{-1}\log t^2 + \bar{P}_2t + \cdots)$, $\bar{P}_1 \sim x'^{\frac{1}{2}}$, $\bar{H}_8'(\xi_{s2}) - x'^{\frac{1}{2}}$,
order: $x'^{-\frac{1}{2}}$.

(H_9). $H_9 = \bar{W}_0 + \bar{W}_{4/3}z^{4/3} + \bar{W}_{5/3}z^{5/3} + \bar{W}_2z^2 + \bar{W}_{7/3}z^{7/3} + \bar{W}_{8/3}z^{8/3} + \cdots$,
$\bar{W}_0 = (-\frac{1}{6})\tilde{U}_0$, $\bar{W}_{4/3} = (\frac{1}{4})\tilde{U}_{1/3}$, $\bar{W}_{5/3} = (\frac{3}{20})\tilde{U}_{2/3}$, $\bar{W}_2 = 0.1\tilde{U}_1$,
$\bar{W}_{7/3} = (-\frac{1}{7})[\bar{W}_{4/3} + (\frac{1}{2})\tilde{U}_{4/3}]$, $\bar{W}_{8/3} = (\frac{1}{24})[\bar{W}_{5/3} + 1.5\tilde{U}_{5/3}]$,
$\bar{W}_3 = (\frac{1}{3})[\bar{W}_2 + (\frac{1}{4})\tilde{U}_2]$, etc., $\tilde{U}_0 = N_1'\epsilon_1A_1^{-2}$, $\tilde{U}_{1/3} = 2N_{4/3}'\epsilon_1A_1^{-2}$,
$\tilde{U}_{2/3} = (\frac{10}{3})(N_1'a_5 + N_{5/3}'\epsilon_1)A_1^{-2}$, $\tilde{U}_1 = 5[N_1'\epsilon_2 + (\frac{10}{9})N_{4/3}'a_5$
$+ N_2'\epsilon_1]A_1^{-2}$, $\tilde{U}_{4/3} = [8N_{4/3}'\epsilon_2 + (\frac{25}{9})N_{5/3}'a_5 + 7\epsilon_1]A_1^{-2}$,
$\tilde{U}_{5/3} = (\frac{1}{3})[28N_1'a_8 + 35(N_{5/3}'\epsilon_2 + N_2'a_5) + 28N_{8/3}'\epsilon_1]A_1^{-2}$,
$\tilde{U}_2 = [12N_1'\epsilon_3 + (\frac{112}{9})N_{4/3}'a_8 + 16N_2'\epsilon_2 + (\frac{40}{9})N_{7/3}'a_5 + 12N_3'\epsilon_1]A_1^{-2}$, etc.,
$\epsilon_1 = [0.2A_1^4 + (\frac{10}{3})A_1^2c_1 + a_3]$, $\epsilon_2 = [-0.84A_1^4 - (\frac{8}{3})A_1^2c_1 + a_6]$,
$\epsilon_3 = (0.29A_1^4 + a_9)$. $\bar{H}_9 = \bar{Z}_{-1}(z-1)^{-1} + \bar{Z}_0 + \bar{Z}_1\log(z-1)^2$
$+ \bar{Z}(z-1) + \cdots$, $\bar{Z}_{-1} = (\frac{1}{6})\bar{X}_{-2}$, $\bar{Z}_1 = (-\frac{1}{3})[\bar{Z}_{-1} - (\frac{1}{2})\bar{X}_{-1}]$,
$\beta_0 = (-\frac{1}{3})[\bar{Z}_1 - (\frac{1}{2})\bar{X}_0]$, $\bar{Z}_2 = 2\bar{Z}_1 - (\frac{1}{2})\bar{X}_1$, $\bar{Z}_3 = (\frac{1}{3})[\beta_2 - (\frac{5}{3})\bar{Z}_2$

$-(\frac{1}{6})\bar{X}_2]$, $\bar{Z}_4 = (-\frac{1}{2})[\bar{Z}_3 + (\frac{1}{18})\bar{X}_3]$, etc., $\bar{X}_{-2} = 3A_1^{-2}P_1''\epsilon_4$, $\bar{X}_{-1} = A_1^{-2}P_1''\epsilon_4$, $\bar{X}_0 = A_1^{-2}[P_1''(4\epsilon_5 + 9\epsilon_6) - P_1'(\epsilon_4 - 3\epsilon_5) - 3P_2'\epsilon_4]$, $\bar{X}_1 = A_1^{-2}[3P_1''(5\epsilon_6 - 6\epsilon_7) + P_1'(5\epsilon_5 + 9\epsilon_6) - P_2'(5\epsilon_4 - 12\epsilon_5) - 9P_3'\epsilon_4]$, $\bar{X}_2 = A_1^{-2}[-3P_1'(4\epsilon_6 - 3\epsilon_7) + P_2'(16\epsilon_5 + 27\epsilon_6) - P_3'(12\epsilon_4 - 27\epsilon_5) - 18(P_4'\epsilon_4 + P_1''\epsilon_7)]$, etc., $\epsilon_4 = (0.81A_1^4 + 2A_1^2c_1)$, $\epsilon_5 = [-0.27A_1^4 - (\frac{8}{3})A_1^2c_1 + c_2]$, $\epsilon_6 = 0.09A_1^4 + c_3$, $\epsilon_7 = 0.1A_1^4 - 2c_4$, $\bar{H}_9(\xi_{e2}) = 0$ if $\bar{Z}_1 = -(\bar{Z}_{-1}t^{-2} + \bar{Z}_0t^{-1} + \bar{Z}_1t^{-1}\log t^2 + \bar{Z}_2t + \cdots)$, $t = t(x)$, $\bar{H}_9'(\xi_{e2}) \sim x'$, $\bar{Z}_1 \sim x'$, order: x'^{-1}.

(H_{10}). $H_{10} = \bar{N}_0 + \bar{N}_1z + \bar{N}_{5/3}z^{5/3} + \bar{N}_2z^2 + \bar{N}_{7/3}z^{7/3} + \bar{N}_{8/3}z^{8/3} + \bar{N}_3z^3 + \cdots$, $\bar{N}_1 = (\frac{1}{4})\bar{M}_0 - 3\bar{N}_0$, $\bar{N}_{5/3} = (\frac{3}{20})\bar{M}_{2/3}$, $\bar{N}_2 = (-0.4)\bar{N}_1$, $\bar{N}_{7/3} = (\frac{1}{14})\bar{M}_{4/3}$, $\bar{N}_{8/3} = (\frac{3}{56})\bar{M}_{5/3} - (\frac{1}{7})\bar{N}_{5/3}$, $\bar{N}_3 = (\frac{1}{6})[\bar{N}_2 + (\frac{1}{4})\bar{M}_2]$, etc., $\bar{M}_0 = (-6)L'$, $\bar{M}_{2/3} = (-12)L_2'$, $\bar{M}_{4/3} = (-7.5)L_4'$, $\bar{M}_{5/3} = (-\frac{3.2}{5})L_5'$, etc., $\bar{H}_{10} = \bar{S}_0 + \bar{S}_1(z-1) + \bar{S}_2(z-1)^2 + \cdots$, $\bar{S}_0 = (\frac{1}{6})\bar{R}_0$, $\bar{S}_2 = 2\bar{S}_1 - (\frac{1}{2})\bar{R}_1$, $\bar{S}_3 = (-\frac{1}{6})[2\bar{S}_2 + (\frac{1}{2})\bar{R}_2]$, $\bar{S}_4 = (-\frac{1}{2})[\bar{S}_3 + (\frac{1}{18})\bar{R}_3]$, etc., $\bar{R}_0 = (-3)(2M_0' + M_1')$, $\bar{R}_1 = (-3)(3M_1' + M_2')$, $\bar{R}_2 = (-3)(2M_2' + M_3')$, $\bar{R}_3 = -(5M_3' + 3M_4')$, $\bar{R}_4 = (-3)[1.5M_4' + M_5']$, etc. B.C. $\bar{H}_{10}(\xi_{e2}) = 0$ if $\bar{S}_1 = -(\bar{S}_0t^{-1} + \bar{S}_2t + \bar{S}_3t^2 + \cdots)$, $\bar{H}_{10}'(\xi_{e2}) = 1.5\bar{S}_1 + \cdots$, $\bar{H}_{10}'(1) \neq 0$, $\bar{S}_1 \sim \bar{S}_0K^{-1}x'^{\frac{1}{3}} + \cdots$, order: $x'^{-\frac{1}{3}}$.

APPENDIX—PART II.

III. 1. $a_3 = (-2)a_0$, $a_5 = (-6.6)\beta_1$, $a_6 = 0.4a_0$, $a_8 = 5.7\beta_1$, $a_9 = 0.04a_6$, etc., $c_2 = 6\beta_1$, $c_3 = (\frac{3.4}{9})\beta_1$, $c_4 = (-\frac{5.9}{81})\beta_1$, $c_5 = (\frac{4.4.9}{13.15})\beta_1$, etc.

III. 3. $\theta_{v21} = m_0 + m_1z + m_2z^2 + m_3z^3 + m_4z^4$, $m_1 = (-2)(2m_0 + A_1^3C')$, $m_2 = [-(1.5m_1 - 1.2A_1^3C']$, $m_3 = [(-\frac{2}{3})m_2 - 0.5A_1^3C']$, $m_4 = [(-\frac{1}{4})m_3 + (\frac{1}{11})A_1^3C']$, $m_0 = -[x'^{\frac{1}{3}}C'A_1i(x') + m_1\xi_{e2}^{3/2} + m_2\xi_{e2}^3 + m_3\xi_{e2}^{9/2} + m_4\xi_{e2}^6]$, $\theta_{v22} = \bar{p}_0 + \bar{p}_1z + \bar{p}_2z^2 + \bar{p}_3z^3 + \bar{p}_4z^4$, $\bar{p}_1 = -(4\bar{p}_0 + \delta_2)$, $\bar{p}_2 = (\frac{1}{4})(\delta_2 - 3\bar{p}_1)$, $\bar{p}_3 = (-\frac{1.8}{8.8})\bar{p}_2$, $\bar{p}_4 = (-\frac{1}{4})\bar{p}_3$, $\delta_2 = 4A_1C'c_1$, B.C. $\theta_{v22}' = 0$ for $\eta_1 = 0$, $\theta_{v22}(\xi) = 0$ for $\xi = \xi_{e2}$ if $\bar{p}_0 = -(\bar{p}_1\xi_{e2}^{3/2} + \bar{p}_2\xi_{e2}^3 + \bar{p}_3\xi_{e2}^{9/2} + \bar{p}_4\xi_{e2}^6)$, $\theta_{v22}'(\xi_{e2}) = \theta_{v22}'(1) + \cdots = 1.5\bar{p}_1 + 3\bar{p}_2 + \cdots = \text{const.}$, order: $x'^{-\frac{1}{3}}$. $\theta_{v23} = E_0 + E_3z + E_5z^{5/3} + E_6z^2 + E_8z^{8/3} + \cdots$, $E_3 = (-6)E_0$, $E_5 = 0.1\delta_4$, $E_6 = 9E_0$, $E_8 = (-\frac{7.8}{8.6.0})\delta_4$, etc., $\delta_4 = 18(3A)^{\frac{1}{3}}A_1C'$. B.C. $\theta_{v23}'(y') = 0$ for $y' = 0$. $\bar{\theta}_{v23} = F_0 + F_1(z-1) + F_2(z-1)^2 + F_3(z-1)^3 + \cdots$, $F_0 = (\frac{3.3.7}{8.8.0})\bar{\delta}_4$, $F_1 = (-\frac{3.4.7}{8.8.0})\bar{\delta}_4$, $F_2 = (-\frac{1.0.5}{7.8.8})\bar{\delta}_4$, $F_3 = (\frac{2.3.3}{8.9.1})\bar{\delta}_4$, etc., $\bar{\delta}_4 = 6A_1(3A)^{\frac{1}{3}}C'$. $\bar{\theta}_{v23}(\xi_{e2}) = 0$ if $F_4 = -(F_0t^{-4} + F_1t^{-3} + F_2t^{-2} + F_3t^{-1} + F_5t + \cdots)$, $t = t(x')$, $\bar{\theta}_{v23}'(\xi_{e2}) = \bar{\theta}_{v23}'(1) + \cdots = 1.5F_1 + \cdots = \text{const.}$, order: $x'^{-\frac{1}{3}}$. $\theta_{v24} = N_0 + N_1z + N_{5/3}z^{5/3} + N_2z^2$

$+ N_{8/3}z^{8/3} + N_3z^3 + \cdots$, $M_0 = A_1C'[0.2A_1^4 + (\frac{10}{3})A_1^2c_1 + a_3]$,
$M_{2/3} = (\frac{10}{3})A_1C'a_5$, $M_1 = 5A_1C'[-0.84A_1^4 - (\frac{8}{3})A_1^2c_1 + a_6]$,
$M_{5/3} = (\frac{28}{3})A_1C'a_8$, $M_2 = 12A_1C'(0.29A_1^4 + a_9)$, etc. $\theta_{v24} = L_0$
$+ L_1(z-1) + L_2(z-1)^2 + \cdots$, $K_0 = -(A_1C')(1.62A_1^4$
$+ 10A_1^2c_1 - 3c_2)$, $K_1 = (-A_1C')[0.54A_1^4 + (\frac{40}{3})A_1^2c_1 - 5c_2 - 9c_3]$,
$K_2 = 0.18A_1^4 + 12c_3$, $K_3 = (-1.1)A_1^4$, etc.

III. 4. $L_4' = (\frac{1}{6})(3A)^{4/3}A_1 - 9(3A)^{-\frac{1}{3}}A_1^4c^2 + (\frac{10}{9})(3A)^{\frac{1}{3}}a_5$,
$L_7' = -[(\frac{1}{2})(3A)^{4/3}A_1 - (\frac{27}{3})(3A)^{-\frac{1}{3}}A_1^4c^2 + (\frac{10}{9})(3A)^{\frac{1}{3}}a_5 + (\frac{8}{3})(3A)^{\frac{1}{3}}a_8]$,
$L_{10}' = 2[(\frac{1}{6})(3A)^{4/3}A_1 + 9(3A)^{-\frac{1}{3}}A_1^4c^2 - \frac{20}{9}(3A)^{\frac{1}{3}}a_8]$,
$L_{13}' = (-4.5)(3A)^{-\frac{1}{3}}A_1^4c^2$.

III. 5. $C_{10} = (-9)(3A)^{-4/3}c^2C'$, $C_{11} = 4A_1C'$, $C_{12} = 4u_0^2\rho_0p_0^{-1}$,
$C_{12} = 6\sqrt{2}(3A)^{-\frac{1}{3}}cu_0^2\rho_0p_0^{-1}$, $N_0' = N_i'$ (Harm) $+ (C_{12}/C_{11})L'$,
$N_i' = (-C_{12}'/C_{10})L_2'$, $N_{2/3}' = (-1.5)(C_{12}'/C_{10})L_4'$, $N_1' = C_{10}^{-1}[C_{11}N_{2/3}'$
$- 1.5C_{12}L_2' - C_{12}'L_5']$, $N_{4/3}' = 0.75(C_{11}/C_{10})\{-N_0' + N_i'[n + (\text{Harm})]$
$+ N_1'\}$, $N_{5/3}' = (\frac{3}{5})C_{10}^{-1}(C_{11}N_{4/3}' - 0.75C_{12}L_4' - C_{12}'L_7')$,
$N_2' = 0.5C_{10}^{-1}[(-N_{2/3}' + N_{5/3}')C_{11} - 0.6C_{12}L_5' - C_{12}'L_8']$,
$N_{7/3}' = (\frac{3}{7})(C_{11}/C_{10})[-N_i'(n + B_2) - N_1' + N_2']$, etc.,
$B_2 = (\frac{1}{2} + 1 + \frac{3}{2} + 2 + \frac{5}{2} + \cdots)$, $B_3 = (\frac{1}{3} + 1 + \cdots)$ etc.,
$P_1' = \{-C_{10}^{-1})(C_{12}M_0' + C_{12}'M_1')$, $P_2' = (-\frac{1}{3})P_1' - 0.5C_{10}^{-1}(C_{11}P_0'$
$C_{12}M_1' + C_{12}'M_2')$, $P_3' = (-\frac{4}{7})P_2' + [(\frac{1}{21}) - (C_{11}/3C_{10})]P_1'$
$- (\frac{1}{6})(C_{12}/C_{10})M_2' - (\frac{1}{3})(C_{12}'/C_{10})M_3'$, etc., $P_0' = -\{P_1't(x')$
$+ P_2'[t(x')]^2 + P_3'[t(x')]^3 + \cdots\}$.

NOTES FROM THE NATIONAL BUREAU OF STANDARDS.*

PHYSICAL CONSTANTS OF LEATHER AND COLLAGEN.

Expanded research at the National Bureau of Standards in the field of leather has added fundamental data on the physical and physicochemical properties of leather and its parent substance, collagen. These constants are largely unknown, even though the art of leather manufacture predates historical records. Published data are lacking on such properties as expansivity, compressibility, dry density, and specific heat. Results of this program, sponsored by the Office of the Quartermaster General, should prove of value to the entire leather industry, whose facilities for research of this nature are limited.

The lack of such data may be attributed directly to the extreme chemical and physical complexity of the system of tanned collagen fibers known as leather. Although many workers are inclined to believe that leather is not a pure substance and is subject to variation because of differences in the hide itself and those introduced by nonuniformity of the tanning process, it is a matter of practical as well as theoretical interest to determine either the values of the physical constants or their orders of magnitude. A knowledge of these constants may permit utilization of leather in applications requiring specific properties possessed by leather but as yet unknown. The effect of tannage and other treatments on values of such constants may yield valuable information concerning the structure of collagen and the mechanism of tanning.

Since leather is a porous material containing as much as 60 per cent void space, most measurements of physical properties of the leather substance present obvious difficulties. One method of solving this problem lies in completely filling the leather with a liquid and ascertaining the properties of the liquid-leather system. This approach was used by Charles E. Weir of the Bureau's Leather Laboratory to obtain data on the cubical expansivity of leather. Using water as the confining liquid, dilatometric measurements of the volume of a water-leather system were made and compared with measurements on a similar system containing water alone.

As a result of these measurements it was found that the average expansivity of 540×10^{-6}/°C. applies for collagen and all leathers except chrome-vegetable leather. The average expansivity of the latter tannage is 340×10^{-6}/°C. These averages have reproducibilities as indicated by standard deviations of 13×10^{-6}/°C. and 37×10^{-6}/°C., respectively.

* Communicated by the Director.

If a system containing leather in water is heated, it is known that above a well defined temperature, depending on tannage and other factors, the leather undergoes a dimensional change commonly called shrinkage. During this process the area decreases drastically while the thickness increases. The temperature at which this process begins is defined as the shrinkage temperature and is considered a measure of the efficiency of the tanning process—the higher the temperature, the more thorough the tannage. During the dilatometric studies it became evident that the apparent shrinkage actually represented an increase in real volume. This increase in volume amounted to approximately 1 per cent and was irreversible in character although the purely thermal expansion was found to be nearly completely reversible.

It was also observed that this increase in real volume occurred very slowly at temperatures well below the shrinkage temperatures of the specimens found by regular test methods. This indication that shrinkage was a rate process was substantiated by a rate measurement which indicated that the expansion occurring during shrinkage follows a first order process.

NOTE: For further technical details see "Effect of temperature on the volume of leather and collagen," by CHARLES E. WEIR, *J. Research*, NBS Vol. 41, p. 279 (1948) (RP1924).

MICROWAVE MEASUREMENT STANDARDS.

As part of the broad program for the establishment of national standards and calibration services for all electrical quantities at radio frequencies, microwave measurement standards are being intensively developed at the National Bureau of Standards in the range from 300 to 100,000 megacycles and above. This work, under the direction of Dr. Harold Lyons, has resulted not only in extremely precise and accurate standards of frequency, power, attenuation, and other quantities, but has also made possible precision measurements in a whole new field of microwave spectroscopy formerly inaccessible to investigation because of the limitations of infrared and optical equipment. Of basic importance in the microwave program has been the development and continued improvement of a primary standard of frequency accurate to one part in 100 million. This standard, based on a quartz-crystal clock and a frequency multiplying system governed by the time observations of the U. S. Naval Observatory, is now being used by the Bureau to provide a regular service to Government and industry consisting of frequency measurements and calibrations of frequency meters and voltage sources.

As a result of research carried on under the pressure of World War II, a vast new spectrum of radio frequencies above 300 megacycles is now available for application to radar, navigation systems, storm and weather reporting, relays for FM and television broadcasting, blind

bombing, guided missiles, and many other uses, both peacetime and military. The fundamental requirement for opening up and fully exploiting this new microwave region, however, is the development of national standards and measurement methods for frequencies up to 100,000 megacycles or more. Such standards are necessary tools for the design, development, and production engineering of practical electronic equipment, as well as for basic research as in nuclear physics. Many of the new techniques in atomic energy work, in particular some of the recent billion-volt nuclear particle accelerators, depend upon accurate microwave measurement methods and equipment. Microwave standards are also important in the development and proper use of a number of industrial and medical applications of radio and electronics, such as dielectric heating for case-hardening metals, rubber curing, plastic molding, food processing, textile fabrication, and radio therapy.

The comprehensive program on microwave measurements at the National Bureau of Standards began in 1944, when the United States Joint Chiefs of Staff requested the Bureau to undertake the development of a microwave standard of frequency. With the assistance of the Massachusetts Institute of Technology Radiation Laboratory, a preliminary standard was developed in 1945 and placed in service for instrument calibration. As the result of continued research in this field at the Bureau, it is now the most complete and accurate primary frequency standard in the world, having an accuracy of one part in 100 million and continuous coverage over the range from 300 to over 40,000 megacycles. The extension of range to the millimeter bands above 30,000 megacycles is now being undertaken and is important for work on microwave spectroscopy, microwave optics, and applications requiring sharp microwave beams of high resolution, such as short-range target-seeking equipment for rockets and guided missiles.

The frequency source for this equipment is a quartz-crystal unit with a series-resonance frequency of approximately 100 kilocycles per second. The national primary frequency standard consists of nine such oscillators which are automatically compared with each other and with corrected U. S. Naval Observatory time. The best oscillators are constant to one part in a billion for short time intervals and drift less than one part in a hundred million per month.

The 100-kilocycle output from the primary standard is first multiplied to 7500 kilocycles and then added in a converter to the output of one of three precision variable oscillators to give an output which is variable from 9.5 to 10.5 megacycles. The output of the converter is passed through five consecutive frequency multipliers with outputs available in the vicinity of 30, 90, 270, 810, and 2400 megacycles. Each of these stages has a tuning range of about 10 per cent so as only to accommodate the range of variable frequency, making it impossible to tune

to any except the proper harmonic. The output from any of the five multiplier stages can be impressed across a nonlinear element, such as a crystal rectifier, which will generate higher-order harmonics.

When continuous frequency coverage is obtained through use of a precision oscillator, the frequency stability of any one of these harmonics is about 95 per cent dependent upon the primary standard and only 5 per cent dependent upon the oscillator. A suitable electronic circuit measures the instantaneous frequency of the oscillators to within one cycle, thus giving an over-all accuracy, for continuous coverage, of one part in ten million.

At times a calibration is needed at several points within a range of frequencies rather than at a particular frequency. To obtain such a series of precisely known frequencies easily, spot-frequency generating equipment is used in which the frequency is at all times dependent directly upon the primary standard. Here the precision oscillators are replaced by a system of multipliers and converters, known as the decade generator, which gives an output adjustable in 100-kilocycle steps from 2.0 to 3.0 megacycles. This is accomplished by adding to a −2.0 megacycle frequency the output of any one of a series of frequency multipliers which provide outputs every 100 kilocycles from 100 to 1000 kilocycles. This frequency is then added to the 7.5-megacycle output and passed through the frequency multipliers as before. Any one of the outputs is now adjustable over an approximately 10-per cent range at intervals of 1 per cent. As these frequencies are derived directly from the primary standard, they are known to within one part in 100 million. Although the output from the microwave frequency standard is tunable over a relatively narrow frequency range, continuous frequency coverage can be obtained through the overlapping of harmonics. With the 10-per cent bandwidth being used, harmonics above the 10th overlap. The output of any one stage may be used up to the 30th harmonic with power sufficient for frequency calibration purposes. To generate these high-order harmonics, the output from any one of the five frequency multiplier stages is impressed across a crystal diode mounted in a coaxial line or a section of wave guide.

To calibrate an unknown signal or frequency meter at frequencies below 2700 megacycles, a superheterodyne receiver is used with a panoramic adapter. For the high frequencies, a spectrum analyzer, consisting essentially of a superheterodyne receiver with a frequency-modulated local oscillator, is employed. Although the standard microwave frequency is known to one part in 100 million, it is practically impossible to certify a frequency meter to this accuracy. Many factors enter into the exact determination of the resonance frequency, limiting the accuracy of the usual meter to about one part in 100 thousand or less.

When a precise determination of the resonance frequency is desired, such variables as temperature, humidity, and pressure may have to be

considered. In addition, the amount of external reactance which is coupled into the frequency meter also affects the resonance frequency and must be taken into account or made negligible during the calibration. Finally, the sensitivity of the resonance-indicating device and the finite Q of the meter itself limit the accuracy to which resonance can be determined.

In addition to work on the microwave frequency standard, the problems involved in the development of microwave standards of power, attenuation, impedance, field intensity, and noise and of dielectric and magnetic measurements are being strenuously attacked at the Bureau. The production of new insulating and dielectric materials for microwave radio equipment requires standard dielectric samples and dielectric measuring equipment, particularly at the upper ranges, where power losses in present materials are serious. The development of microwave optics, allowing the use of lenses, prisms, and other optical elements in radio, will be advanced when suitable materials are found. Radomes for radar sets also urgently require improved materials. Techniques have therefore been developed and equipment constructed for dielectric measurements in the 1000-, 3000-, 9000-, and 24,000-megacycle ranges. Both resonant-cavity and impedance methods are used. In the first method the dielectric sample is placed in a resonant cavity and the change in frequency and Q measured, while in the second the impedance of a section of line containing the sample is determined. Other procedures are also under investigation. For example, a microwave refractometer has been completed which continuously records the index of refraction or dielectric properties of atmospheric air. This new instrument will aid materially in FM, television, and radar propagation studies since the index of refraction along the propagation path determines the course of a radio wave and hence its range.

The problem of a primary standard of attenuation capable of measurements at all microwave frequencies has been very satisfactorily solved with the design of a new waveguide-below-cutoff primary standard of higher accuracy than has previously been attained. This standard is needed for signal-generator, power, and voltage calibrations of transmitters, receivers, and other equipment so that, for example, the range of communication and radar transmitters can be kept up to the proper operating levels.

Attenuators are calibrated to read power ratios directly in decibels and are used to control the power level of microwave signals in any system such as a receiver, signal generator, or power meter. Because of the tremendous range of the microwave spectrum over which attenuators must be calibrated, the design of a series of standard microwave attenuators to cover the entire spectrum and the use of the direct substitution method of calibration are not practicable. A technique for use in calibration at any frequency was therefore worked out at the

National Bureau of Standards by development of the heterodyne or intermediate-frequency substitution method. This method permits the comparison of an unknown attenuator, operating at an arbitrary frequency, with the standard attenuator operating in the intermediate-frequency channel of a superheterodyne detecting and measuring receiver. Thus, one standard attenuator, operating at a convenient frequency, may be used to calibrate attenuators over the entire microwave spectrum.

Essentially, the heterodyne method operates as follows: A radio-frequency generator feeds power through an unknown attenuator into a linear frequency converter, which converts the microwave frequency into the intermediate frequency. The converter feeds through an intermediate-frequency standard attenuator into an amplifier followed by a detector and meter. The unknown attenuator then is removed, and the standard attenuator adjusted to give the same meter reading. The attenuation of the unknown is then equal to the increase of attenuation of the standard.

This method assumes that the frequency converter is linear, that is, that the intermediate-frequency power from the converter is proportional to the input microwave power. For small enough input power this is true for a crystal converter, and the operating conditions for a prescribed degree of linearity may be determined experimentally.

The theoretical attenuation constant of this standard attenuator, which can be calculated by electromagnetic theory, has been checked experimentally to better than 0.01 decibel. Linearity of the attenuator is secured by highly symmetric construction and the use of mode filters. A new, improved version of the attenuator using an electro-formed, rectangular waveguide will continuously cover a range up to 100 decibels and will be temperature-insensitive and free from mechanical wear. Mechanical accuracy of better than 0.001 decibel is obtained, and electrical accuracy approaching this figure is expected through the use of the rectangular waveguide construction, and improved measurement methods.

Accurate power meters are needed for dosage measurement in radiation therapy as well as in the measurement of the intensity of spectral lines and the power of transmitters. Development and construction of the most precise microwave power measuring console in existence has been largely completed; theoretical and experimental work has also been carried out on bolometer mount losses which for the first time threw light on the accuracy of bolometer power meters as primary, absolute standards. An investigation of thermal noise sources as power standards has begun, and ultrasensitive calorimeters are being planned for precision power measurements of high absolute accuracy.

Research is continuing at the Bureau having as its object the extension of microwave standards for all electrical quantities to higher frequencies. In the frequency standard, higher powers than ever before

have been generated at the upper end of the range by means of a new light-house tube frequency multiplier recently devised at the Bureau. By harmonic generation, it should be possible, with the development of suitable crystal rectifier mounts, to obtain useful outputs to 100,000 megacycles and above. These extremely high frequencies may require the introduction of new frequency measuring instruments similar to optical apparatus.

Development of invariant primary and secondary frequency standards using spectrum lines of gases is also well under way. Absorption cell techniques and methods have been formulated by the use of atomic beams in which the resonance frequencies of individual atoms serve as standards for time-keeping and frequency measurements. The extension in frequency range is making possible work on microwave spectroscopy as well as applications to equipment development using millimeter waves. Such short waves permit the use of sharp microwave beams with small, light antennas.

Spectroscopic analysis has hitherto been dependent on infrared, optical, and ultraviolet methods, which for the most part are limited to work on atoms and the simpler molecules. However, a large part of medical and industrial chemistry requires analyses of large, complicated molecules. The recent advances in microwave measurement techniques, opening up the new field of microwave spectroscopy, now provide methods of unprecedented accuracy and resolution for the study of new atomic and molecular properties. This spectroscopic research, now under way at the Bureau, is also yielding a basically new, primary standard of frequency and time that is invariant with age. Specifically, a preliminary model of an atomic clock has been developed using spectral lines. The atomic clock method promises to surpass the accuracy of the rotating earth as a time keeper by one or two orders of magnitude.

In order to apply microwave spectroscopy to standards and measurements work, new wide-range search and precision measurement spectrometers have been designed and are under construction. Work has also begun on the measurement and compilation of spectrum lines as secondary frequency standards. These investigations are being extended below the frequency range of presently available lines and later will be extended far up into the millimeter bands.

The Bureau's research in microwave spectroscopy has resulted not only in an atomic clock and frequency standard but also in the development of stabilized oscillator-frequency multiplier chains locked to a spectrum line by means of a servo-type electronic circuit. The Bureau's work has also shown that directly controlled oscillators analogous to low-frequency quartz-crystal oscillators can also be built in which an absorption line replaces the quartz crystal. Such circuits will have extensive application to frequency measurement and control of transmitters for FM and television relays, communications, and radar.

THE FRANKLIN INSTITUTE.

ANNUAL MEETING, WEDNESDAY, JANUARY 19, 1949.

The Annual Meeting of The Franklin Institute was held on Wednesday evening, January 19, 1949. Mr. Richard T. Nalle, President, presided. There were approximately 225 persons in attendance. The President stated that the minutes of the Stated Monthly Meeting for December were printed in full in the January issue of the Journal and if there were no corrections or additions the minutes were to stand approved as printed. There was no contrary motion.

The President then announced that the annual report from the Board of Managers, due to be presented at this meeting, would be presented at an adjourned meeting in the Spring.

The Secretary gave his report of Membership for the month of December, which was as follows:

Active	32
Associate	24
Student	15

The total Institute Membership as of December 31, 1948, was 5436.

The Secretary announced the report of the Tellers on the annual election for members of the Board of Managers who are to serve for a period of three years. They are as follows:

Rupen Eksergian
W. H. Fulweiler
Horace P. Liversidge
Malcolm Lloyd, Jr.
Marshall S. Morgan
Edward Warwick
Edward G. Budd, Jr.
William J. Meinel

The President then declared the above members duly elected to the Board of Managers. The Tellers, Professor Charles Gay, Dr. George S. Crampton, and Mr. Penrose Hoopes, were thanked for their efforts on behalf of the Institute in counting the ballots.

The Secretary then announced the Science is Fun Lecture, to take place on February 10th, which will open the exhibit of the Baltimore and Ohio Railroad model.

The legacy of $2,000 given to The Franklin Institute in memory of the late Alexander E. Outerbridge was announced by the Secretary.

The President presented Dr. W. H. Worrilow, President of the Lebanon Steel Foundry Company of Lebanon, Pennsylvania, who spoke on "Foundries—Past and Present; Their Transition From Manual to Mechanical Operation." Dr. Worrilow outlined the historic sequence of the foundries industry from its inception to the present time of mechanized operation. His talk took the form of a narrative rather than a technical discussion on how foundries emerged from manual to mechanical production. He discussed modern methods, their applications, and their uses. Dr. Worrilow further referred to the development of the making of intricate iron and aluminum castings.

The meeting was adjourned with a rising vote of thanks to Dr. Worrilow for his excellent paper.

HENRY B. ALLEN
Secretary.

COMMITTEE ON SCIENCE AND THE ARTS.

(Abstract of Proceedings of Stated Meeting held Wednesday, January 12, 1949.)

HALL OF THE COMMITTEE,
PHILADELPHIA, JANUARY 12, 1949.

DR. JULIAN W. HILL in the Chair.

The following report was presented for final action:

No. 3193: Fawick Airflex Clutch.

This report recommended the award of a John Price Wetherill Medal to Thomas L. Fawick, of Cleveland, Ohio, "In consideration of the adaptation of the well-known qualities of rubber for expansion, flexibility and durability, to its use in the form of an inflatable tire or bag between two rotating members to act as a clutch, and its use between a rotating member and a stationary member to act as a brake, and the excellent design and construction of the details developed, resulting in a practical device, widely and successfully used in industry."

JOHN FRAZER,
Secretary to Committee.

ACTIVE MEMBERS ELECTED AT THE MEETING OF THE BOARD OF MANAGERS JANUARY 19, 1949.

ACTIVE FAMILY.

Walter J. Cantley
R. C. Dove
George Willauer, M.D.

ACTIVE.

Joseph D. Anastasi, M.D.
James Arnao, M.D.
W. C. Bobbitt
A. J. Cascaden
Joseph T. Devlin, M.D.
Joseph A. Donnelly
William G. Everhart
J. Arthur Fanning
Wilbur W. Ferris
P. J. Gambescia, M.D.
J. Gershon-Cohen, M.D.
W. D. Gillen
Raphael H. Greenstein, M.D.
Arthur A. Hartley, M.D.
Julius Jacobson
Roger B. Jones
C. Blaine Lytle
James T. Murphy
R. C. Osborne
Ernest R. Schmidt
Edward Louis Schwartz
Morton A. Serrell
Robert Z. West
George S. Wheeler

ACTIVE NON-RESIDENT.

J. H. van Santen

NECROLOGY.

William H. Burkhardt '46
Edward D. Chaninel '46
Frederick A. Depue '43
Mrs. G. W. C. Drexel '34
Joseph L. Schwartz '44
Herbert J. Tily, Mus.D., LL.D., L.H.D., '24
Lewis H. Van Dusen '36
William F. Willcox '80
Richard D. Wood '34

BY-LAWS.

ARTICLE I.

STOCK.

Section 1. The Real and Personal Estates of the Institute as held upon the First day of January, One Thousand Eight Hundred and Eighty-one, shall be valued at One Hundred Thousand Dollars, and shall be represented by Ten Thousand Shares of Stock of the par value of Ten Dollars each. Said shares shall be divided into two classes, *viz.*:

First Class. Shares not registered for use: on which no annual payment shall be charged or collected, and the holders thereof shall not have the privileges of members of the Institute, but may, if of legal age, vote at any annual election for managers upon the payment of One Dollar upon each share of stock on which they may desire to vote; provided, however, such shares have been held by the same person at least three months before such election.

Shares of the First Class may be converted into shares of the Second Class at the pleasure of the owners, provided the transfer be approved by the Board of Managers; but, when once so converted, they shall always continue in the Second Class.

Second Class. Shares registered for use: on which Twelve Dollars per annum shall be due and payable from resident members in advance on the first day of October in each year, except as hereinafter provided.

Non-resident holders of Second Class stock shall pay an annual fee of Five Dollars.

Section 2. The holders of Second Class stock shall be entitled to the use of the library, lectures and reading-room, and, if of legal age, to all other privileges of membership in the Institute, so long as they make the annual payment in advance; and shall, on the payment of One Dollar therefor, be entitled to a Certificate of Membership.

Section 3. If the annual dues for successive years remain unpaid at the expiration of two and a half years on any share of stock of the Second Class, such shares shall then become forfeited to the Institute; but such forfeiture may be remitted by a unanimous vote of the Board of Managers.

Section 4. Stock of the Second Class may be held in trust for persons not of legal age, and shall be liable to the payment of only one-half the annual fees due upon stock of Second Class held by persons of legal age; provided, that when such minors arrive at legal age, new certificates, subject to the full annual contribution, shall issue on payment of the customary fee.

Section 5. Certificates for the First Class stock may be issued for any number of shares in a single certificate; but every certificate for the Second Class shall be for one share only.

Section 6. No share of stock in the Second Class shall be transferred until all arrearages and fines are paid, and all books and tickets returned, and the transfer approved by the Board of Managers.

Section 7. All certificates of stock shall be signed by the President and Secretary; shall be issued by the Controller, and shall be transferable only on the books of the Institute by the owner, or his legal representative, on the surrender of the old certificate, and of a fee of twenty-five cents for each certificate.

Section 8. All subscriptions to stock shall be approved by the Board of Managers before the certificate can be issued.

ARTICLE II.

MEMBERS.

Section 1. All persons interested in the purposes and activities of the Institute and willing to further them may become members when elected by the Board of Managers, or in a manner prescribed by the Board, except as qualified by Section 6 of this ARTICLE. Membership shall consist of the following classes:

1. Student members
2. Associate members
3. Active members
4. Sustaining members
5. Honorary members

The Board of Managers may establish other classes of members, provided that the privileges enjoyed and dues paid by such other classes are not inconsistent with the privileges and dues of the classes of members specifically provided for by these By-Laws.

Section 2. Student members shall be under twenty-five years of age and shall pay annual

dues of $2.00. They shall be entitled to unlimited free admission to the Museum and the Planetarium. Student members who are fourteen or more years of age may, if endorsed by a teacher or an Active member, have the use of the library upon the payment of additional annual dues of $1.00 but shall have no voting privileges or rights to hold office.

Section 3. Associate members shall pay annual dues of $5.00. They shall be entitled to unlimited free admission to the Museum and to the Planetarium, but shall have no voting privileges or rights to hold office. Upon the payment of $5.00 additional annual dues they shall have Family privileges as defined in Section 7 of this ARTICLE.

Section 4. Active members shall be not less than twenty years of age. They shall pay annual dues of $15.00. Active members, residing permanently at a distance of fifty miles or more from Philadelphia, shall pay annual dues of $7.50. Active members shall be entitled to use the library, to receive one copy of the Journal of The Franklin Institute, to vote and to hold office. They shall be entitled to all the privileges of Associate members and upon the payment of $5.00 additional annual dues shall be entitled to Family privileges.

Section 5. Sustaining members shall be not less than twenty years of age. They shall pay annual dues of not less than $50.00. They shall be entitled to all of the privileges of Active members and shall have Family privileges as defined in Section 7 of this ARTICLE free of additional charge.

Section 6. Honorary members shall be entitled to all of the privileges of sustaining members, except the right to vote and to hold office. They shall be nominated by the Board of Managers and shall be elected by four-fifths of the votes of the members present at any stated meeting of the Institute at which their nomination may be acted upon.

Section 7. Family privileges consist of the right to receive a card for each individual in the family of the member and resident with the member entitling the holder to unlimited free admission to the Museum and the Planetarium.

Section 8. Members belonging to classes of membership existing prior to the amendment of this ARTICLE shall be reclassified in a manner consistent with their former dues and privileges.

ARTICLE III.

PAYMENT OF DUES.

Section 1. The annual fees for membership shall be due and payable on the first day of each month, whichever is nearest to the date of election or as determined by the Board of Managers.

Section 2. Any member whose dues are more than two months in arrears shall have all the privileges of membership suspended until such time as all arrears are paid. Should the dues not be paid when they become six months in arrears the said member shall forfeit his membership.

Section 3. The Board may remit temporarily in whole or in part the dues of any member either by action in a particular case or by establishing regulations governing certain cases.

Section 4. Every person admitted to membership in the Institute shall be considered as liable for the payment of dues until he shall have resigned, been dropped or have been relieved therefrom by the Board of Managers.

Section 5. Resignations of memberships shall be made to the Board of Managers in writing, but need not be accepted until all dues and arrears up to date of resignation shall have been paid.

Section 6. The privileges and title of Associate member may be obtained for life by paying therefor in one year the sum of $100. From this payment may be deducted one-half of the Student or Associate Membership dues paid by the member during the preceding ten years, but in no case shall the deduction exceed $50.00.

Section 7. The privileges and title of Active Member shall be enjoyed for life or may be obtained for life by a member who has heretofore or who shall hereafter pay therefor, in one year, the sum of $300, except that a person residing permanently at a distance of twenty-five miles or more from Philadelphia may become an Active Member for life by paying therefor in one year the sum of $100. From this payment may be deducted one-half of the dues paid by the member during the preceding ten years, but in no case shall the deduction exceed one-half of said payment.

Associate Members for life may become Active Members for life by paying therefor in one year the sum of $200.

Section 8. Firms, corporations, associations or individuals may nominate and subscribe for the membership dues of groups of members of any class or classes, at the annual dues provided for, subject to the approval of the Board as to any particular nominee. If the dues of these nominees amount to $100 or more in the aggregate, the firm, corporation, association or individual shall be known as an Affiliate of the Institute.

ARTICLE IV.

MANAGEMENT.

Section 1. The Institute shall be governed by a board of twenty-four (24) Managers elected by the members.

Section 2. The officers, who shall be elected by the Board of Managers, shall be a President, an Executive Vice President, not more than five Vice Presidents, a Secretary, an Assistant Secretary, a Treasurer, and an Assistant Treasurer. The Board of Managers may elect such other officers as it deems necessary.

Section 3. At the annual meeting of the Institute, eight Managers shall be elected each year to serve for three years, provided that the Managers now elected, or who may hereafter be elected, shall continue to serve until their successors be appointed.

Section 4. All elections of the Institute shall be by letter ballot and no vote may be cast by proxy.

Section 5. Nominations for Managers shall be made in writing at the stated meeting in the month of December. Each nomination paper must be signed by at least two members, who shall certify that the candidate will serve if elected. After the nominations are closed, the President shall appoint three members, who are neither officers nor nominees, to act as tellers of election. The list of nominees shall be posted at the Institute and incorporated (with directions for voting) in a ballot to be sent to each member by the Secretary at least one week before the date of election. Each ballot shall be accompanied by a return envelope addressed "To the Tellers of Election," and provided with a space for the signature of the member voting.

Section 6. On the date of the annual meeting, and at an hour previously designated by their chairman, the tellers shall meet at the Institute, and shall count all legal votes that have been received by mail or placed in the ballot box before eight o'clock P.M.; and when the count is completed they shall report to the annual meeting of the Institute the total number of ballots cast, together with the number of votes received by each candidate. Thereupon the presiding officer shall announce the names of the candidates who received the plurality of votes, and shall declare them elected Managers of the Institute for the ensuing terms.

Section 7. At the organization meeting of the Board of Managers, the Board of Managers shall elect the officers provided for in Section 2 of ARTICLE IV to serve for one year; and may at said organization meeting or from time to time thereafter elect such other officers as it may determine upon, and shall determine and fix the compensation, if any, to be paid to the officers so elected or appointed by them. The officers who are in office immediately prior to the annual meeting shall continue in office until their successors are elected or appointed by the Board of Managers as herein provided.

Section 8. Vacancies occurring in any office may be filled by the Board of Managers by election or appointment of persons to serve until the next annual election.

ARTICLE V.

BOARD OF MANAGERS.

Section 1. The Board of Managers shall have general charge and control of the Institute and of the Benjamin Franklin Memorial and The Franklin Institute Museum, and shall consist of twenty-four members elected as provided in ARTICLE IV. The President, the Executive Vice President, the Vice Presidents, the Secretary, the Treasurer, the Chairman of the Committee on Science and the Arts, and the Chairman of the Library Committee shall be ex officio members. A quorum of the Board of Managers shall be nine of the elected and ex officio members. The Board of Managers may adopt such by-laws, rules and regulations for the governance of their affairs as are not inconsistent with the Charter and these By-Laws.

Section 2. They shall present, through the President, at the annual meeting of the Institute, a report of the condition of the affairs of the Institute.

Section 3. They shall hold stated meetings in each month except in July and August.

Section 4. Special meetings may be called by the President at his discretion and shall be called by him on written request of the Executive Committee or of any seven members of the Board. In case of his absence or refusal to act, such special meeting shall be called by the Secretary.

Section 5. Members who have not attended five regular meetings in the twelve months prior to the stated meeting of the Institute in December, shall be reported thereat as having resigned, unless it be unanimously voted by the Board at that meeting that such member has been absent for sufficient reason.

Section 6. All vacancies on the Board of Managers shall be filled by the Board until the next annual meeting of the Institute.

Section 7. The Board of Managers may elect an Executive Committee consisting of five of its Members with such authority to act on behalf of the Board as the Board may delegate.

ARTICLE VI.

DUTIES OF OFFICERS.

Section 1. The President shall be the executive head of the Institute and, under the supervision of the Board of Managers, shall have general charge of the affairs of the Institute. He shall preside at all meetings of the Institute and of the Board of Managers and shall be ex-officio a member of all standing committees of the Institute.

Section 2. The Executive Vice President, under the supervision of the President, shall have immediate charge of all affairs of the Institute other than those matters specifically delegated by action of the Board of Managers pursuant to these By-Laws to the Executive Committee and the Finance Committee. He shall prepare the annual budget, and submit it to the Board of Managers for approval. He shall be an ex-officio member of all standing Committees of the Institute. He shall report to the President. In case of the disability of the Executive Vice President, the President shall designate another officer or other officers to assume his duties.

Section 3. In the absence of the President, the Executive Vice President shall exercise his duties. In the absence of both the President and the Executive Vice President, the Vice Presidents shall exercise the duties of President in order of their seniority in office.

Section 4. The Secretary shall be responsible for keeping the minutes of all meetings of the Institute and of the Board of Managers. shall keep the records of the Institute, and shall perform all the duties usually pertaining to the office of Secretary. He shall report to the Executive Vice President. In the absence or disability of the Secretary, the Assistant Secretary shall perform his duties.

Section 5. The Treasurer shall have custody of all monies received from the Committee on Finance together with all monies received by the Institute from dues, admissions, and other operations of the Institute, depositing them, in the name of the Institute, in such bank or banks as the Board of Managers shall direct and disbursing them by checks signed as the Board of Managers shall likewise direct. He shall keep accounts of the receipts and disbursements and shall report thereon to the Board of Managers as directed. He shall have general supervision of the accounts of the Institute and shall render such financial statements as directed by the Board of Managers. He shall give bond to an amount and with such surety as the Board of Managers shall determine. The Treasurer shall report to the Executive Vice President. In the absence of the Treasurer, the Assistant Treasurers shall in the order of their seniority in office, perform his duties.

ARTICLE VII.

BOARD OF COUNCILLORS.

The Board of Councillors shall consist of not more than fifty members and shall embrace, as ex-officio members, the Governor of Pennsylvania, the Chief Justice of the Supreme Court of Pennsylvania, the Mayor of the City of Philadelphia, the President of the City Council of Philadelphia, the President of the Board of Public Education and the Superintendent of Schools of the City of Philadelphia, the Provost and the President of the University of Pennsylvania, the President of Commissioners of Fairmount Park,

the President of the American Philosophical Society, the President of the Poor Richard Club, and the President and Secretary of The Franklin Institute, which two latter shall respectively be Chairman and Secretary of the Board of Councillors. Additional members shall be nominated by the President or Secretary of the Institute and elected by the Board of Managers, at such intervals as the Board of Managers may see fit. The Board of Councillors shall meet at the call of its Chairman or of the Board of Managers for the consideration of any matter that pertains to the welfare of the Institute. The Secretary shall from time to time send to the members of the Board of Councillors a report of the activities and accomplishments of the Institute.

ARTICLE VIII.

COMMITTEES OF THE INSTITUTE.

Section 1. There shall be the following Standing Committees of the Institute:

1. Bartol Research Foundation Committee.
2. Biochemical Research Foundation Committee.
3. Committee on Science and the Arts.
4. Endowment Committee.
5. Finance Committee.
6. Library Committee.
7. Meetings Committee.
8. Membership Committee.
9. Museum and Memorial Committee.
10. Publications Committee.
11. Committee on Research.

Section 2. Each standing committee of the Institute shall consist of the number of members determined on by the Board of Managers and composed of members of the Board of Managers and/or non-members as may be desirable, except that the Committee on Science and the Arts shall consist of not less than sixty nor more than seventy-five members, and except that the Bartol Research Foundation Committee shall consist of not less than ten nor more than fifteen members as provided for in Section 7 of this ARTICLE.

Section 3. The members of each committee shall serve one year, except that members of the Committee on Science and the Arts shall serve three years.

Section 4. The members of each committee, other than ex-officio members specifically provided for by these By-Laws, shall be appointed by the President and approved by the Board of Managers at the organization meeting of the Board following the annual meeting, except that each year but one-third of the total number of members of the Committee on Science and the Arts shall be so appointed and approved.

Section 5. The Chairman of each Committee shall be designated by the President, except that the Library Committee and the Committee on Science and the Arts shall select their chairmen. A chairman who is not an elected or ex-officio member of the Board of Managers shall have the privilege of the floor at meetings of the Board but shall not have the right to vote.

Section 6. Each Committee shall report monthly to the Board of Managers through its Chairman. A copy of their report shall be in the hands of the Executive Vice President one week before the monthly meeting of the Board.

Section 7. The Bartol Research Foundation Committee shall consist of not less than ten nor more than fifteen members, including the President and the Executive Vice President of the Institute and the Director of the Bartol Research Foundation as ex-officio members and nine members of the Institute of whom not more than four shall be members of the Board of Managers. The Committee shall, after consultation with the Director of the Bartol Research Foundation and subject to the approval of the Board of Managers, collaborate with the Executive Vice President in carrying out the purposes and determining the policies of the Bartol Research Foundation. It shall appoint the Director and the Staff of the Laboratories, subject to the approval of the Board of Managers. The Director of the Bartol Research Foundation, to whom all other employees of the Foundation shall report, shall have charge of the scientific activities of the Foundation and shall report on all matters to the Committee or to the Executive Vice President, as the Committee may elect.

Section 8. The Biochemical Research Foundation Committee shall report to the Board of Managers from time to time on the condition of the affairs and operation of said Foundation and shall be available to the

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Director of the Biochemical Research Foundation and its Advisory Council for collaboration and advice.

Section 9. The Committee on Science and the Arts shall investigate current discoveries, inventions and other achievements in the sciences and their application in the mechanical and industrial arts with a view of affording such recognition as may lie within the power of the Institute to bestow. The Committee shall exercise independent judgment therein. In all matters of a general or administrative nature the Committee shall collaborate with the Executive Vice President.

Section 10. The Endowment Committee shall, in collaboration with the Executive Vice President, encourage the gift of funds to the Institute.

Section 11. The Finance Committee shall have the custody and control of all the securities and investments of the Institute with full power and authority to buy and to sell, and to invest and reinvest the same; including the power to satisfy mortgages and extinguish ground rents, and to direct the placing of all such insurances on Real Estate held for investment as it may deem necessary, and to make such improvements, repairs and alterations to such Real Estate as it may deem necessary. It shall have power to authorize the proper officers of the Institute to execute the necessary papers to effect all purchases, sales and assignments of property other than real estate; to transfer registered securities; to subscribe to bond-holders' agreements to plans of reorganization involving any securities held by the Institute or in which it has an interest; and to do all such acts as are necessary in pursuance of the foregoing powers. It may, with the approval of the Board of Managers, appoint a Trust Company of the City of Philadelphia to act as Fiscal Agent under the direction of the Committee.

The Finance Committee shall keep a record of all its acts and proceedings, which shall be communicated to the Board of Managers.

Section 12. The Library Committee, in collaboration with the Executive Vice President, shall be of counsel and advice in the purchase of books and publications suitable to and consonant with the purposes of the Library; and in other matters pertaining to the Library.

Section 13. The Meetings Committee shall, in collaboration with the Executive Vice President, secure for presentation before the Institute papers dealing authoritatively with subjects of import in the fields of science, engineering and industry.

Section 14. The Membership Committee shall, in collaboration with the Executive Vice President, promote the several memberships defined in ARTICLE II of these By-Laws.

Section 15. The Museum and Memorial Committee shall, in collaboration with the Executive Vice President, be of counsel and advice to the Director of the Benjamin Franklin Memorial and The Franklin Institute Museum, and with the scientific staff of the Museum concerning the activities and the exhibits of both the Museum and the Memorial.

Section 16. The Publications Committee, in collaboration with the Executive Vice President, shall be of counsel and advice with regard to all publications of the Institute other than publications of the Bartol Research Foundation.

Section 17. The Committee on Research shall, in collaboration with the Executive Vice President, be of counsel and advice to all research activities of the Institute except those of the Bartol and Biochemical Foundations.

Section 18. Each committee shall organize and adopt rules as it sees fit, subject to the provisions of these By-Laws and the approval of the Board of Managers.

ARTICLE IX.

MEETINGS.

Section 1. The Institute shall hold stated meetings on the third Wednesday of each month except in June, July, August and September, at 8:15 P.M.

Section 2. Special meetings shall be called by the President upon the order of the Board of Managers or within ten days upon the written application of twenty-four voting members of the Institute.

Section 3. Twenty-four members shall constitute a quorum at any stated or special meeting.

Section 4. The Annual Meeting of the Institute shall be held on the third Wednesday of January of each year at 8:15 P.M.

Elections at Annual Meetings shall not result from the poll of less than forty-eight ballots cast in accordance with ARTICLE VI, Section 6, of these By-Laws.

ARTICLE X.

AMENDMENTS.

These By-Laws may be altered or amended at any stated meeting of the members of the Institute, provided notice in writing, signed by two members, of the proposed alteration or amendment, shall be given to the Board of Managers two months prior to the said meeting, except that amendments to ARTICLE I, relating to capital stock, must be ratified subsequently by a majority of the stock represented at a meeting specially called for this purpose.

MUSEUM NOTES.

When The Franklin Institute was first organized at the beginning of the nineteenth century, the founders decided to provide a cabinet of minerals. This was the modest beginning of the present museum. The reputation of the Institute which gradually expanded as its activities increased, induced the owners to confide to the Institute a few notable museum exhibits of historic or technical interest. At the time when these objects came into our possession the prevailing ideas of museums had not grown beyond the stage of regarding them as repositories.

The present Franklin Institute Museum as envisaged by the late Dr. McClenahan viewed the subject from a different aspect. The museum was to be regarded as a proved instrument of educational exposition—that the existence of a museum technique was more important than the chance accumulation of objects gathered over a long period of time. The Franklin Institute Museum as it is now administered may be regarded as creating the objects in its collections through the art of idealogical arrangement. Its rapid growth in material strength and public favor demonstrates that it has overcome the timeworn prejudice that a museum exists as a custodian of things.

The general purpose of the museum is to display objects which will show the growth and development of science and its application to industry, assuming that every section of the population has an equal interest in technological science because all alike are consumers of the products of scientific culture. The museum is designed to give visitors fascinating illustrations to deepen their understanding of specific matters not within their knowledge.

Mechanical science is only a curious toy until it has been clearly related to human affairs. A chemical process, for instance, has little significance to an uninformed spectator when shown as an isolated event. The same process, if demonstrated as a step in the manufacture of some article in common use, possesses a greater degree of meaning. When the process is further related to social events by illustrating its effect upon employment and the utilization of new ideas, educational values are brought out that are applicable to the whole community.

Special attention is devoted to the graphic arts and to those phases of science in which Benjamin Franklin earned distinction. The historical display of printing provides a fascinating voyage in an ingenious discovery and is romantically relevant to an institution which is dedicated to the life and work of the great Franklin.

While the best museum techniques available within the Institute's means are developed and applied, care is taken to avoid creating an atmosphere suggestive of an intellectual opiate. Every endeavor is made to create a feeling of friendliness and comfort. Such a purpose can only be achieved by the active co-operation of industry, and it has been the fortunate experience that exhibitors are making a marked distinction between exhibits of a trade-show nature and those with a meaning for non-technical visitors. It will be acknowledged that the exhibits now being furnished by industrial exhibitors are of high educational value and are tastefully displayed.

THE FRANKLIN INSTITUTE LABORATORIES FOR RESEARCH AND DEVELOPMENT.

The following are abstracts of papers printed in technical journals other than the JOURNAL OF THE FRANKLIN INSTITUTE, but written by members of the staff of The Franklin Institute Laboratories for Research and Development.

A Spiral Chronograph for Long Time Intervals (*The Review of Scientific Instruments*, Vol. 19, No. 7).—A spiral chronograph has been developed for measuring the time between any pair of a series of transient events that occur within a period of 12 milliseconds. The maximum error in the measurement of such an elapsed time is ± 6 microseconds.

The chronograph differs basically from those of Emrich and Moran as follows: (1) it measures time periods up to 12 milliseconds; (2) a logarithmic spiral is developed; (3) the spiral is generated by a circuit which avoids a modulating network.

The spiral trace is presented on the screen of a cathode-ray tube. The trace is modulated by a crystal-controlled oscillator to provide accurate time markers. Each transient event produces a momentary radial deflection of the spiral trace. Elapsed time between any two events is determined by counting the time markers between the corresponding signals.

The accuracy of determining elapsed time between signals is primarily limited by errors involved in reading the photographic records since the absolute error in crystal frequency is less than 0.01 per cent. When using 10-microsecond time markers and a sweep speed of 2 milliseconds per revolution, each signal may be located within ±3 microseconds so that the maximum error in measuring elapsed time between signals is ±6 microseconds. Using the same time markers, the peripheral distance between them will increase as the sweep speed is increased, thereby resulting in increased reading accuracy. This will be gained at the expense of the total time of duration of the spiral. H. D. WARSHAW.

High Accuracy Contour Cams (*Product Engineering*, August, 1948).—A contour or tape-wheel cam is a logarithmic spiral shape. Pairs of contour cams rotate without sliding; a condition of pure rolling. Recently they have been used in computing and control equipment. In these, they have been applied for certain computing problems requiring great accuracy.

Unlike other cam and follower combinations, contour cams are reversible; hence when one function is mechanized, its inverse may be obtained. Although contour cams are difficult to make and cannot be used for nonmonotonic functions, their use is justified by their distinct advantages.

The author develops, in the article, the basic formulas for the contour cams, also those for the points of inflection which may occur. In addition, the formulas are given for the center of the cutter used for cutting such cams. An example is developed to illustrate the applications of the formulas.

There is an admitted difficulty in designing and cutting contour cams; but their use for computations may be justified by the superior accuracy that is possible.

Friction problems are not critical in contour cams since the cam shafts may rotate in ball bearings. It appears that a combination of many pairs of contour cams in a more complicated computing problem would still yield satisfactory results, with little cumulative lost motion or other adverse effects.

In the problem solved here, addition and subtraction are accomplished by the use of differentials; multiplication and division are performed by means of addition and subtraction, using a logarithmic scale on contour cams. Although this type of cam is suitable only for limited types of function, nonmonotonic functions being eliminated, it has the distinct advantages of being quite accurate, and of being reversible. R. O. YAVNE.

LIBRARY NOTES.

The Committee on Library desires to add to the collections any technical works that members would wish to contribute. Contributions will be gratefully acknowledged and placed in the library. Duplicates received will be transferred to other libraries as gifts of the donor.

Photostat Service. Photostat prints of any material in the collections can be supplied on request. The average cost for a print 9 × 14 inches is thirty-five cents.

The Library and reading room are open on Mondays, Tuesdays, Fridays, and Saturdays from 9 A.M. until 5 P.M., and Wednesdays and Thursdays from 2 P.M. until 10 P.M.

RECENT ADDITIONS.

AERONAUTICS.

COURANT, RICHARD AND FRIEDRICHS, K. O. Supersonic Flow and Shock Waves. 1948.
NORTHRUP AERONAUTICAL INSTITUTE. Aircraft Power Plants. 1948.

AGRICULTURE.

HERRINGTON, B. L. Milk and Milk Processing. 1948.

ARCHITECTURE AND BUILDING.

TURNER, ARTHUR W. AND JOHNSON, ELMER J. Machines for the Farm, Ranch and Plantation. 1948.

BIOGRAPHIES.

EATON, JEANETTE. That Lively Man, Ben Franklin. 1948.
POSIN, DANIEL Q. Mendeleyev. 1948.
RUSH, BENJAMIN. An Autobiography. 1948.

CHEMISTRY AND CHEMICAL TECHNOLOGY.

FINDLAY, ALEXANDER. A Hundred Years of Chemistry. Second Edition. 1948.
HERSTEIN, KARL M. AND JACOBS, MORRIS B. Chemistry and Technology of Wines and Liquors. Second Edition. 1948.
HEVESY, GEORGE. Radioactive Indicators. 1948.
HIMUS, GODFREY W. The Elements of Fuel Technology. 1947.
KOBE, KENNETH A. Inorganic Process Industries. 1948.
MCILROY, ROBERT J. The Chemistry of the Polysaccharides. 1948.
MITCHELL, JOHN J. AND SMITH, DONALD MILTON. Aquametry. 1948.
READ, JOHN. Humour and Humanism in Chemistry. 1947.
SCHMIDT, ALOIS X. AND MARLIES, CHARLES A. Principles of High Polymer Theory and Practice. 1948.
YOUNG, RONALD S. COBALT. 1948.

ELECTRICITY AND ELECTRICAL ENGINEERING.

BROUGHTON, H. H. Electric Winders. Second Edition. 1948.
MASON, WARREN P. Electromechanical Transducers and Wave Filters. Second Edition. 1948.

ELECTRONICS.

HORNUNG, J. L. Radar Primer. 1948.
MARCUS, JOHN AND ZELUFF, VIN. Handbook of Industrial Electronic Circuits. 1948.
POLLARD, ERNEST C. AND STURTEVANT, JULIAN M. Microwaves and Radar Electronics. 1948.
WESTINGHOUSE ELECTRIC CORP. ELECTRONICS ENGINEERS. Industrial Electronics Reference Book. 1948.

ENGINEERING.

BUCHAN, S. Rubber to Metal Bonding. 1948.
KACHANOW, L. N. AND OTHERS. Plastic Deformation. 1948.
ZENER, CLARENCE. Elasticity and Anelasticity of Metals. 1948.

FOOD.

MARK, E. M. AND STEWART, GEORGE, Ed. Advances in Food Research. Volume 1. 1948.

GEOLOGY.

BULLEN, K. E. An Introduction to the Theory of Seismology. 1947.
COTTON, CHARLES ANDREW. Climatic Accidents in Landscape Making. 1942.
SHEPARD, FRANCIS P. Submarine Geology. 1948.

INDUSTRIAL MANAGEMENT.

SCHUTT, WILLIAM H. Process Engineering. 1948.
TRUNDLE, GEORGE T. AND OTHERS. Managerial Control of Business. 1948.

MATHEMATICS.

BOCHNER, SALOMON AND MARTIN, W. T. Several Complex Variables. 1948.
CARATHEODORY, CONSTANTIN. Vorlesungen über Reelle Funktionen. 1948.
CHILD, J. M. Elements of Coordinate Geometry. 1933.
HAHN, HANS AND ROSENTHAL, ARTHUR. Set Functions. 1948.
KLEIN, CHRISTIAN FELIX. Elementary Mathematics from an Advanced Standpoint. 1945.
MURNAGHAN, FRANCIS D. Introduction to Applied Mathematics. 1948.
SCHELKUNOFF, S. A. Applied Mathematics for Engineers and Scientists. 1948.
SOMMERVILLE, DUNCAN M. Y. Analytical Geometry of Three Dimensions. 1947.
USPENSKY, JAMES VICTOR. Theory of Equations. 1948.
WEATHERBURN, C. E. Differential Geometry of Three Dimensions. Volume 1–2. 1930–1947.

MECHANICAL ENGINEERING.

FAIRES, VIRGIL MORING. Theory and Practice of Heat Engines. 1948.
NORMAN, C. A. AND ZIMMERMAN, R. H. Introduction to Gas Turbine and Jet Propulsion Design. 1948.

METALLURGY.

KIEFFER R. AND HOTOP, W. Sintereisen und Sinterstahl. 1948.

MILITARY ENGINEERING.

HATCHER, JULIAN S. The Book of the Garand. 1948.

PHOTOGRAPHY.

AMERICAN SOCIETY OF MAGAZINE PHOTOGRAPHERS. Photographic, 1949. 1948.
BAKER, T. THORNE. Photographic Emulsion Technique. Second Edition. 1948.

PHYSICS.

EVANS, RALPH M. An Introduction to Color. 1948.
HARRISON, GEORGE R.; LORD, RICHARD C. AND LOOFBOUROW, JOHN R. Practical Spectroscopy. 1948.
HECTOR, LUTHER GRANT; LEIN, HERBERT S. AND SCOUTEN, CLIFFORD E. Physics for Arts and Sciences. 1948.
KRAUSKOPF, KONRAD BATES. Fundamentals of Physical Science. 1948.
LAMB, HORACE. Hydrodynamics. Sixth Edition. 1945.
LAPP, R. E. AND ANDREWS, H. L. Nuclear Radiation Physics. 1948.
MCKAY, HERBERT C. Principles of Stereoscopy. 1948.
MOTT, NEVELL FRANCIS AND SNEDDON, IAN N. Wave Mechanics and its Applications. 1948.
NEWMAN, F. H. AND SEARLE, V. H. L. The General Properties of Matter. Fourth Edition. 1948.
ROBERTSON, JOHN KELLOCK. Radiology Physics. Second Edition. 1948.

STREETER, VICTOR L. Fluid Dynamics. 1948.
TIMOSHENKO, STEPHEN AND YOUNG, D. H. Advanced Dynamics. 1948.

PLASTICS.

DUNHAM, ARTHUR. Working with Plastics. 1948.

RADIO.

BARLOW, H. M. Micro-waves and Wave Guides. 1947.
RADIO Component Handbook. 1948.

SANITARY ENGINEERING.

PARKER, MILTON. Food Plant Sanitation. 1948.

SCIENCE.

SWEZEY, KENNETH M. After Dinner Science. 1948.

TEXTILES.

HIND, JOHN RICHARD. Woollen and Worsted Raw Materials. Second Edition. 1948.

PUBLICATIONS RECEIVED.

Practical Spectroscopy, by George R. Harrison and others. 605 pages, 14 × 22 cm., tables, drawings and illustrations. New York, Prentice-Hall, Inc., 1948. Price, $6.65.

Introduction to Engineering Problems, by Robert Q. Brown. 191 pages, 14 × 22 cm., drawings. New York, Prentice-Hall, Inc., 1948. Price, $3.95.

Microwaves and Radar Electronics, by Ernest C. Pollard and Julian M. Sturtevant. 426 pages, 14 × 22 cm., drawings and tables. New York, John Wiley & Sons, Inc.; London, Chapman & Hall, Ltd.; 1948. Price, $5.00.

Gas Tables, Thermodynamic Properties of Air Products of Combustion and Component Gases, by Joseph H. Keenan and Joseph Kaye. 238 pages, 19 × 26 cm., tables. New York, John Wiley & Sons, Inc.; London, Chapman & Hall, Ltd.; 1948. Price, $5.00.

Advanced Dynamics, by S. Timoshenko and D. H. Young. 400 pages, 15 × 23 cm., tables and drawings. New York, McGraw-Hill Book Co., Inc., 1948. Price, $5.50.

Fluid Dynamics, by Victor L. Streeter. 263 pages, 15 × 23 cm., drawings. New York, McGraw-Hill Book Co., Inc., 1948. Price, $5.00.

Evaluation of Residual Stress, by K. Heindlhofer. 196 pages, 14 × 21 cm., drawings, tables and illustrations. New York, McGraw-Hill Book Co., Inc., 1948. Price, $4.00.

A Source Book in Greek Science, by Morris R. Cohen and I. E. Drabkin. 579 pages, 16 × 23 cm., drawings. New York, McGraw-Hill Book Co., Inc., 1948. Price, $9.00.

Velocity-Modulated Thermionic Tubes, by A. H. W. Beck. 180 pages, 15 × 22 cm., drawings and tables. New York, The Macmillan Co.; Cambridge, The University Press; 1948. Price, $3.75.

Principles of Radar, by Denis Taylor and C. H. Westcott. 141 pages, 15 × 22 cm., plates, drawings and tables. New York, The Macmillan Co.; Cambridge, The University Press; 1948. Price, $3.50.

A Hundred Years of Chemistry, by Alexander Findlay. 318 pages, 14 × 22 cm. New York, The Macmillan Co.; London, Gerald Duckworth & Co., Ltd.; 1948. Price, $4.50.

Principles of Stereoscopy, by Herbert C. McKay. 191 pages, 15 × 24 cm., drawings and illustrations. Mass., American Photographic Publishing Co., 1948. Price, $5.00.

American Photographic Annual of Photography 1949, Edited by Frank R. Fraprie and Franklin I. Jordan. 240 pages, 18 × 25 cm., illustrations and drawings. Mass., American Photographic Publishing Co., 1948. Price, $2.00 (Paper).

Mechanical Vibrations, by William Tyrrell Thomson. 222 pages, 14 × 22 cm., drawings and tables. New York, Prentice-Hall, Inc., 1948. Price, $5.00.

Chemistry and Uses of Insecticides, by E. R. De Ong. 345 pages, 15 × 23 cm., illustrations and tables. New York, Reinhold Publishing Corp., 1948. Price, $6.00.

Science at War, by J. G. Crowther and R. Whiddington. 185 pages, 14 × 22 cm., drawings and illustrations. New York, Philosophical Library, 1948. Price, $6.00.

Combustion Engines, by Arthur P. Fraas. 439 pages, 15 × 23 cm., tables, drawings, illustrations and plates. New York, McGraw-Hill Book Co., Inc., 1948. Price, $5.50.

James Watt and the Industrial Revolution, by H. W. Dickinson and H. P. Vowles. 57 pages, 15 × 22 cm., drawing and illustrations. New York, Longmans, Green & Co., Inc., 1948. Price, 60¢ (Paper).

Theory of Groups and Its Application to Physical Problems, by S. Bhagavantam and T. Venkatarayudu. 234 pages, 14 × 22 cm., drawings. Andhra University, Waltair, India, 1948. No Price.

Michigan and the Cleveland Era, Edited by Earl D. Babst and Lewis G. Vander Velde. 372 pages, 15 × 24 cm., Port. The University of Michigan Press, Ann Arbor, 1948. Price, $2.50.

BOOK REVIEWS.

THEORY AND PRACTICE OF HEAT ENGINES, by Virgil Moring Faires. 388 pages, 16 × 24 cm., tables, drawings, and illustrations. New York, Macmillan Co., 1948. Price, $5.00.

The student of thermodynamics is usually confronted by a great variety of books presenting many phases of the subject ranging from the derivations of the basic equations of classical thermodynamics to the practical mathematics for the steam engineer. Unless the student expects to specialize in this field, it will be physically impossible to devote more than just enough time to the study of the broad field of heat power to obtain a fundamental idea of the theory and practice with reference to heat engines. Books of the type of the subject work are admirably adapted to give a preliminary course in thermodynamics to be followed by more advanced studies, or to provide a review of the subject for the practicing engineer as well as a handy reference work as problems arise. The topics covered are those thought to be most frequently useful to engineers.

The discussion of heat engines starts from the perfect gas law and shows the various effects of changes in pressure, temperature, volume and relative specific heats of the working substance. The relations between energy and heat as inputs are shown, and the resulting outputs in useful work are derived. The general energy equation is shown to be based on the law of the conservation of energy together with its transformation, its transfer and its storage. The methods of calculation of specific values of potential and kinetic energy and flow work are described. Then follows the evaluation of the change in internal energy, of the transferred heat and of the useful work produced. The various thermodynamic processes are then studied and equations developed for each, together with graphical representations in the PV and TS plane showing such phenomena as compression, expansion, change in entropy and transfer of heat.

After this general presentation of basic thermodynamics to the student, specific applications are made of these fundamental laws to such heat engines as compressors, internal combustion engines, steam generators, steam turbines, reciprocating steam engines, refrigeration units and other power equipment. The second law of thermodynamics and its implications are given, showing that available energy is continuously being degraded to unavailable energy; and that all actual processes are irreversible and entropy, like time, is always increasing. The special thermodynamic principles applying to each type of prime mover or heat engine are given, the actual engines described, and the basic design principles explained. Comparisons are freely made among various types of engines and specific designs as to efficiency, operating characteristics and construction advantages. A multitude of examples are solved and many practical problems are given so that the student or engineer can become thoroughly familiar with applications to problems likely to be encountered in every-day work.

The field covered by this book is tremendous and much care has been used to include all the essential engineering facts on heat engines without, at the same time, making it merely a tabulation of data, or neglecting to explain thoroughly all of the basic principles in such detail that the student can easily grasp them.

FRANK R. SIMPSON.

SOURCES OF ENGINEERING INFORMATION, by Blanche H. Dalton. 109 pages, 16 × 24 cm. Berkeley, Calif., University of California Press, 1948. Price, $4.00.

The present volume is aimed at supplying the need for a guide to the source materials of engineering comparable to those guides available for chemistry. Its primary arrangement is by type of material in five main chapters—I. Indexes to periodical and serial literature; II. Abstracts; III. Location of articles; Identification of periodicals; IV. Bibliography; V. Reference books. These chapters are in some cases subdivided into smaller categories and in most cases each is further subdivided by subject. Two other brief chapters discuss "The Trade Catalog Collection" and "Standards and Specifications."

It is apparent that this arrangement is a result of the text having been used as a syllabus in a course on the use of the library, where such a presentation is a logical one. However, for the practicing engineer, who will be primarily interested in subject, the arrangement is not very satisfactory, especially since no index has been provided. For example, this reviewer found the heading "Aeronautics" used eight times and at least three other references to aeronautical material under general headings. Similarly, the heading "Mining" was found eight times. To find all these references necessitates practically a page-by-page perusal of the book. Even if the user is interested only in the strictly bibliographical tools such as indexes, abstracts and bibliographies, there are five lists he will have to consult.

The chapter on abstracts could have been made considerably more useful by listing all abstracts in one list instead of in three sections—"Abstracts, digests in journals and indexes with annotations," "Abstract sections in journals," and "Abstracts in monographs." If indications of the varying types were deemed necessary, it could have been secured by the use of suitable symbols.

Despite the extensive proofreading which is claimed for the text, there seems to be a rather undue proportion of errors and inconsistencies, only a few of which will be noted. On page 7, 1943 is not the last issue of *Geophysical Abstracts*, as it has continued to appear, the last issue being for 1948. The *Experiment Station Record* is given on page 5 as current, but it suspended publication with 1946. Various other publications which have closed are given as current. For example, on page 2 the last issue of Special Libraries Association, Committee on Illumination, *Bibliography of Illumination*, was that of 1929/30. The following entry appears on page 2: "U. S. National Advisory Committee for Aeronautics. Bibliography of Aeronautics. Washington, D. C. 1909+," and on page 24, the same entry appears with the dates 1909–1936, the latter being correct. On page 45 a typographical error makes Bennett appear as Gennett.

It is difficult to criticize such a list as this for omissions, as the compiler may have always had reasons for omitting a particular title. However, in view of other material included, one might suggest the following titles as among those worthy of inclusion: Institute of Gas Technology. *Gas Abstracts*. Chicago 1, 1945+; West, Clarence J. *Bibliography of Pulp and Paper Making*. 1900/28—1936/45. New York, 1929–1947; Bureau of Railway Economics. *Railway Economics. A Collective Catalogue of Books in Fourteen American Libraries*. Chicago, 1912.

In the section Reference Books—General, there are included a number of directories which would seem more logically to belong under the heading Directories. And the value of *Thomas' Register of American Manufacturers* for trade names ought to be noticed.

There is no doubt that this volume contains much valuable information and for checking purposes in libraries it should be a clue to needed material. However, the arrangement and lack of an index hinder its ready use and seem to leave unfulfilled its expressed aim of providing a short cut to research materials.

G. E. PETTENGILL.

INORGANIC PROCESS INDUSTRIES, by Kenneth A. Kobe. 371 pages, 16 × 24 cm., tables, drawings and illustrations. New York, The Macmillan Co., 1948. Price, $6.00.

This book is an elementary survey of inorganic chemical industries with particular emphasis on their historical development and present status and the economic forces affecting chemical processes. It attempts "to supply the basic information needed by the student who must prepare himself to learn from current periodicals what is occurring in the inorganic chemical industry." As the author, Dr. Kobe, Professor of Chemical Engineering at the University of Texas, points out in the preface, no one can keep up with the rapid pace of today's chemical industry without continual reading of current chemical literature.

The first chapter outlines the various types of chemical literature available. The subsequent nine chapters cover industries producing natural sodium salts, natural potassium salts, sodium carbonate, electrolytic caustic and chlorine, chamber, sulfuric acid, contact sulfuric acid, nitrogen fixation, nitrogen products, and phosphorus products. At the end of each chapter on the different industries are student exercises involving the preparation of solubility studies, flow sheets, specifications, patent, economic and market surveys, design of equipment and chemical engineering reports on various processes and equipment. A well selected list of references for recommended reading concludes each chapter. Another feature of the book is the attention given to corrosion problems. The book is up to date. Such recent developments as the vertical mercury electrolysis cell and the commercial preparation of chlorine from salt and nitrogen dioxide are discussed.

This book is not a complete survey of the present inorganic chemical industry. Important topics, processes, and chemicals are omitted. For example, the important and interesting sodium silicates are mentioned only as being involved in the production of silica and vanadium oxide catalyst gels. The 1945 production of sodium silicates was 70 per cent greater than that of the sodium phosphates which are discussed in 8 pages and 23 references. Nothing is given in the book on the glass, ceramic, inorganic pigment, electro thermal or abrasive industries or silver, strontium or barium salts. The treatment of the important problems of water supply and waste disposal is limited to a brief discussion of the use of phosphate for this purpose.

The discussion of the topics presented in the book is very good. The book could be read profitably by those actively engaged in the inorganic chemicals industry as well as by students and others seeking to learn more about this field. Properly supplemented by lectures or outside reading on the topics not covered, the book would be a good text and reference book for a course in inorganic chemical technology.

R. C. MERRILL.

THE ELEMENTS OF PHYSICS, by Alpheus W. Smith. Fifth Edition, 745 pages, 16 × 23 cm., drawings, illustrations and plates. New York, McGraw-Hill Book Co., Inc., 1948. Price, $4.50.

This excellent text-book has been one of the leaders in the field of college physics for more than twenty-five years. It is well known for its wealth of illustrative material, both descriptive and pictorial, as well as for the simplicity and clarity of its presentation of fundamental concepts.

The new fifth edition now appears with added material in meteorology, aero-dynamics, fluid flow, the electron microscope, the cyclotron, the betatron, the synchrotron, and the Van de Graaff high voltage generator. The chapters on electronics, electromagnetic waves and nuclear physics have all been rewritten and brought up to date.

Students who are required to purchase, at considerable cost, and carry around, at considerable inconvenience, a 745-page book such as this do not appreciate the difficulty of the problem faced by the author who tries to keep his text within manageable bounds and at the same time to include a brief description of the highlights of modern physics. Just think what has happened to physics during the past twenty-five years! But the time allotted to a college course in General Physics is still only one year! The poor author must sail his ship between Scylla and Charybdis with small chance of avoiding both.

The material in Professor Smith's chapters on modern physics is good, but there is too much of it. To make room for it too little of the old material has been eliminated. The expedient of relegating many of ihe derivations to an appendix where few students will ever study them cannot be recommended. Such weight as this procedure saves is insignificant. A policy of removing all unessentials, strictly carried out, would result in the elimination of all tables and appendices as well as the chapter on Astrophysics, fascinating as that is, thereby saving forty pages. Another sixty pages would have to be eliminated piecemeal rather than en bloc.

It is almost impossible to revise a text-book without making a slip here and there, such as mentioning a beta-particle 140 pages before telling what such a particle is; or mentioning magnetic susceptibility without ever defining the term. Figure 443 shows the lines of force between like charges and not between unlike charges as stated. Figure 421 shows a Taylor Instrument Company wet- and dry-bulb hygrometer upside down!

FREDERIC PALMER.

A SOURCE BOOK IN GREEK SCIENCE, by Morris R. Cohen and I. E. Drabkin. 579 pages, 16 × 23 cm., drawings. New York, McGraw-Hill Book Co., Inc., 1948. Price, $9.00.

The series of "Source Books in the History of the Sciences," under the general editorship of Dr. Gregory D. Walcott, comprises volumes which are notable contributions to the history of astronomy, mathematics, physics, and geology. Each has dealt with an individual field of science and has been restricted to selections from those writers who are generally credited with having laid the foundations of modern science. The latest addition to the series, in which I. E. Drabkin and the late Morris Cohen treat with Greek science, attains the high standard set by its predecessors.

The purpose of the volume was to correct the common assumption that natural science had its origin in the seventeenth century. Anyone who wishes to know (and all science students should have that wish) what a Greek philosopher said, and not what some scholar says he said, should cherish this volume of selections in translation from the riches of the legacy left by the Greeks.

All the separate streams of knowledge in the ancient world converged on Greece where they were filtered, purified, and turned into more profitable channels by the genius of the race which was first in Europe to arise from obscurity. The Greeks introduced the practice of using argument deliberately as an instrument for discovering the truth. Basing their argument on truths already established, they carried it forward on sound methods of reasoning to a logical conclusion, however unwelcome or unexpected it might be. For this they have long been honored. But it is not so generally understood that they were keen observers and intelligent experimenters. Indeed, they have been accused of neglecting experiment, an accusation which is not sustained by the generous number of selections offered in this volume.

There are some outstanding histories of Greek science but none can convey an adequate picture of the field unless the reader sees something of the original sources upon which the historian relies. Although special fields, like astronomy, botany, and mathematics have been well treated, the reader with more general interests has been compelled to search for source material in various books, not all of which are easily accessible.

For the first time we have in English a generous selection of the fundamental passages from Greek and Latin sources conveniently brought together. Some of these appear in English for the first time. The selections cover ideas in mathematics, astronomy, physics, chemistry, physiography, geography, biology, medicine, and physiological psychology.

The editorial work will meet with nothing but approval. In order to assist the reader to evaluate the historical significance of the selections, the editors have furnished brief introductory comments to each science, and copious explanatory notes and bibliographies.

The book should be open on the knee whenever the student is seeking enlightenment on the growth of one of the sciences with which it treats.

T. C.

CYBERNETICS—OR CONTROL AND COMMUNICATION IN THE ANIMAL AND THE MACHINE, by Norbert Wiener. 194 pages, 16 × 24 cm. New York, John Wiley & Sons, Inc.; Paris, Hermann et Cie; 1948. Price, $3.00.

The transfer of information has for some time been a topic of primary importance to the communications engineer and of comparatively minor concern to many of the other scientific professions. But with the tremendous interest in the past eight years in the practical design and construction of large scale computing machines and gun-fire control equipment, not to mention renewed interest in the newer methods used in the communications industry, there has been slowly developing a unification of thought among the many scientific professions which have to do in any way with the general problems of the transmission of information, especially where the useful information may be accompanied by unwanted distortion or noise. To date little has been written attempting to tie together many of the common problems of data transmission, in the broad sense of these words, as may be of importance to psychologists, physiologists, electrical engineers, mathematicians, and physicists, to name a few. Now in the volume "Cybernetics—or Control and Communication in the Animal and the Machine," the author attempts to break down scientific isolationism by removing some of the previously existing barriers by showing the applications of statistical mechanics methods to communications engineering and control mechanisms such as: electro-mechanical servo-mechanisms; mathematical calculators; automatic control devices; the nervous system; and the brain of the human body.

The fact that there exist recognizable common problems results in a natural attempt to replace or interchange the study of human control functions and electro-mechanical systems. The central theme in the study is the fundamental notion of "the message," whether this be transmitted by electrical, mechanical, or nervous system means. That this is basic will be recognized at once, and it will be further realized that the nervous system acts in many ways as a predicting mechanism, operating on messages or information received in the past. Attempts to mechanize the prediction process were and are still being investigated for application in many fields, especially in designing mathematical calculators and gun-fire control equipment.

The author presents in this book some of the concepts of data prediction based upon knowledge of a message in the presence of noise. Starting with a historico-philosophical discussion of mathematical groups, statistical mechanics, invariants, and ergodic theory, there follow chapters devoted to time series, information, and communication, with some examples chosen from previously published work by the author in the fields of gun-fire control prediction, wave filter theory, his earlier general studies on the subject of Fourier transforms in the complex plane, and feedback and oscillation as these subjects apply to the many interrelated fields noted earlier. Of further interest, especially to the designers of large scale numerical computing machines, is the discussion comparing data transmission, storage, and decision making for the case of such machines and our own nervous system.

This reviewer found those sections which are of a philosophical or descriptive nature to be written in a style conducive to easy but concentrated reading. Those sections, however, which are mathematical in nature, require perhaps a too extensive background in higher mathematical theory for a complete understanding, and in several sections equations and theories are presented with no adequate introduction. Several typographical errors were noted in these mathematical sections.

In spite of these minor criticisms, however, the field of control and communication theory has profited by the publication of this book, which is highly recommended for general reading by physical and even social scientists.

S. CHARP.

RADIO AT ULTRA-HIGH FREQUENCIES, VOLUME II (1940–1947), edited by Alfred N. Goldsmith, Arthur F. Van Duck, Robert S. Burnap, Edward T. Dicky, and George M. K. Baker. 485 pages, 15 × 23 cm., tables, drawings, and illustrations. *RCA Review*, Radio Corp. of America, RCA Laboratories Division, Princeton, N. J., 1948. Price, $2.50.

Radio at Ultra-high Frequencies, Volume II is a collection of papers and abstracts of papers prepared by RCA authors on practically all phases of ultra-high frequency generation, trans-

mission and reception. These papers are reprints of material which has appeared in periodicals since 1940. The work is well arranged and indexed in one volume. Since the material presented in the book is not new, its technical content needs no comment, having previously withstood the test of publication.

As to the arrangement of the book, the material appears under seven headings entitled "Antennas and Transmission Lines," "Propagation," "Reception," "Radio Relays," "Microwaves," "Measurements and Components," and "Aids to Navigation."

In addition to the text proper, the book contains an excellent ultra-high frequency bibliography dating back to 1925, and a section containing summaries of the papers which appeared in Volume I of *Radio at Ultra-High Frequencies*, which is now out of print.

ROBERT S. GRUBMEYER.

THE SPECIFICATIONS AND LAW ON ENGINEERING WORKS, by Walter C. Sadler. 493 pages, 14 × 22 cm. New York, John Wiley & Sons, Inc.; London, Chapman & Hall, Ltd.; 1948. Price, $5.00.

The Specifications and Law on Engineering Works is a well written, well coordinated treatise on those elements of the law with which every engineer should be familiar if he is to protect himself and his employer from those legal entanglements arising primarily in the preparation and interpretation of engineering specifications.

The book is not intended as a substitute for a good lawyer, but rather as a guide to enable the engineer to handle the more elementary legal problems himself and to develop in him sufficient appreciation of the ramifications of the law to enable him to recognize the need for legal assistance when that need exists.

A wide range of legal subject matter is covered. The author treats not only specifications, but also many other phases of contracts. He goes quite thoroughly into the entire legal background of contractual instruments. He also covers the legal aspects of the partnership and corporation. There are several chapters devoted to the history of law which furnish good background as to the underlying theory and concepts of present-day business law.

ROBERT S. GRUBMEYER.

THE MATHEMATICAL SOLUTION OF ENGINEERING PROBLEMS, by Mario G. Salvadori. 245 pages, 16 × 23 cm., tables. New York, McGraw-Hill Book Co., Inc., 1948. Price, $3.50.

Engineering schools have been the target of much criticism by their graduates because the practical applications of mathematics were not sufficiently stressed. This book seeks to bridge the gap between theoretical mathematics and the techniques of solving physical problems by mathematical methods.

Mario G. Salvadori, who is Associate Professor of Civil Engineering at Columbia University, has prepared this excellent text on elementary engineering mathematics. About one-third of the book is devoted to a complete review of the fundamental concepts of mathematics from an intuitional point of view. All that is necessary for an understanding of the book is a knowledge of college mathematics, including the calculus. There are no departures into advanced theories.

The book is outstanding in that a simple physical problem first motivates the introduction of each mathematical technique and of the corresponding theory; the solution of the problem is then carried to its final numerical result, with special emphasis on accuracy of computation and practical numerical schemes. This method of approach is very attractive to the student, since his interest in a subject is greatly enhanced when it is evident how important it is in his field of work. However, the book is not only valuable to students, but also to practicing engineers requiring a review course in mathematics or a wider mathematical background in connection with their work.

The six chapters of the book deal with (1) the fundamental ideas of mathematics; (2) the use of plane Cartesian geometry; (3) the solution of algebraic and transcendental equations; (4) the solution of systems of simultaneous linear equations; (5) the elementary functions of a

real and of a complex variable and power series expansions; and (6) the Fourier series expansion and harmonic analysis.

Kenneth S. Miller, consulting mathematician, has gathered and solved more than 1000 problems which are included in the book, of which about two-thirds are practice problems and one-third applied problems. The applied problems are taken from the various fields of engineering, physics, and mechanics, but do not require a knowledge of these subjects beyond the pre-engineering level.

Professor Salvadori's book is an important contribution to the rapidly advancing field of applied mathematics, and would make an excellent text for such a course on the senior or graduate levels.

D. M. BIBERMAN.

CRYSTAL RECTIFIERS, by Henry C. Torrey and Charles A. Whitmer. 443 pages, 15 × 23 cm., drawings and photographs. New York, McGraw Hill Book Co., Inc., 1948. Price, $6.00.

This is Volume 15 in the Massachusetts Institute of Technology Radiation Laboratory Series. It is a highly professional treatment of a specialized subject. Edited by four staff members, it presents the work of many experts working in fields related to crystal development.

The crystal rectifier, similar in its operation to the antiquated "crystal set" detector, became during World War II one of the key microwave components essential to the realization of radar. It is understandable, therefore, why a tremendous amount of effort was expended by universities and manufacturers for its improvement, and likewise, why a full volume of the Radiation Laboratory Series was devoted to presenting the fund of knowledge so achieved.

All aspects of the development of the crystal are covered in this book. The theory of its operation, techniques of manufacturing and the difficult task of specifying methods of testing and standardization are treated.

As a text this book is outstanding because, like other Radiation Laboratory volumes, it contains material published for the first time which cannot be obtained from any other source. Original copies of Radiation Laboratory reports are becoming dispersed so widely that they are virtually unobtainable except through microfilm service, which likewise is not available to most engineers. Editing of the material published in the reports has made it far more complete and more accurate than it was in its original form.

The current interest in semiconductors which stems from their spectacular applications (typified by the invention of the transistor) places this book in a position of importance in a field other than that in which it was conceived. For, the crystal materials are semiconductors of a highly refined type whose careful preparation reveals techniques of interest to those who would have occasion to study or carry on work in this new and important field. The use of assemblies of semiconductors to replace more complex circuit components such as vacuum tubes may soon become very important to the communications and electronics industries.

Besides the comprehensive treatment of the general properties of semiconductors and metal contacts, the engineer will find an orderly presentation of the latest concepts of the mechanisms for the generation of various types of noise. Since crystals are used in mixer circuits at microwave frequencies, a section is devoted to frequency conversion using the non-linear element. To complement the section on mixer crystals an extremely complete treatment of low-level detection is given. This, like much of the other material, is not to be found in any other publication.

C. W. HARGENS.

SIMPLIFIED DESIGN OF STRUCTURAL TIMBER, by Harry Parker. 218 pages, 13 × 20 cm., tables and drawings. New York, John Wiley & Sons, Inc.; London, Chapman & Hall, Ltd.; 1948. Price, $3.25.

Wood is a commonly used building material. And in these times when much of it is of inferior quality, it is well to learn or review the fundamental design principles so that adequate buildings may be constructed.

This book by Mr. Parker is intended to serve architects, builders, and other interested persons who wish to acquire a knowledge of structural timber design. It is well suited for home study or class room use. To accomplish these aims, the first chapter starts in with the growth of trees and physical properties of wood. The way in which these properties affect the use and grading of lumber is explained.

The next six chapters explain the different stresses to which a building material is subjected. Engineering and building trade terms are clearly defined, and the only mathematics required to understand this book is high school algebra and arithmetic. Only when moment of inertia is discussed does higher mathematics aid in understanding the theory, and here a basic explanation is given for those who do not understand the calculus.

The remaining chapters deal with the design of beams, joints, flooring and other specific types of construction. Bolted joints, timber connectors, and glued joints are described. When modern timber connectors are used, it is possible to develop the full working stress of the timber at the joints, a feat not possible before.

Nearly every type of design problem is illustrated by a numerical example and home work problems. The author develops the basic design principles. Generally he then illustrates the use of tables which enable the designer to choose rapidly the proper size timbers. Forty-five tables are incorporated in this book to eliminate references to other works. Most of these tables are easily used, but Table 3, "Working Stresses for Joists and Planks, Beams and Stringers, Posts and Timbers," would be less confusing if the symbols for the different stresses were omitted. These symbols do not correspond with those used in the formulas in the text.

Nevertheless, the author has accomplished his purpose. This pocket size book is easy to read and understand, convenient to use, and covers the subject from the elementary principles to complicated truss and compound beam design.

E. A, MECHLER.

THE STUFF WE'RE MADE OF, by W. O. Kermack and P. Eggleton. 356 pages, 13 × 19 cm., tables and illustrations. New York, Longmans, Green & Co.; London, Edward Arnold & Co.; 1948. Price, $3.00.

In this second edition of a very well written and easily read book, the authors attempt to guide the reader through some of the fields of the science of Biochemistry, with the objective to survey such parts of this comparatively new science as may be of interest to the general reader who wishes to broaden himself somewhat in this science. Biochemistry has aptly been described as occupying a place between the animate and inanimate worlds of the physical sciences. That this is so can be seen by an examination of some of the subjects considered in this volume. These include the study of: atomic and molecular particles and the physical laws by which they are governed; molecular structure of organic compounds such as proteins and the amino acids and their relationships to the living cells; the chemistry of living tissues; the regulatory mechanisms of the body; dietary factors and vitamins and their relationships to man's growth and well being; sex and reproduction; and chemical makeshifts in nature.

Two objectives were in the minds of the authors: (1) to present a reasonable picture, on the basis of current available knowledge, of the relationship between the living and the non-living; (2) to present an account of the more recent advances in the field of biochemistry and to indicate the significance which these new discoveries may have in our future well being. It is felt that these objectives were attained in a manner which the layman can readily understand and appreciate, although certain sections may not be written to the complete satisfaction of experts in the particular fields.

This book is recommended as general reading material for undergrauate students in the physical sciences and for use by discussion groups interested in the problems associated with the commonly quoted question "Mechanism or Vitalism—which?"

S. CHARP.

NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION.

Electrical Impulse Counter.—JAMES M. GARNER, JR. AND JUNE PETERSON OAKLEY. An impulse counter has been constructed to equip the Biochemical Research Foundation with a counter which combines a number of desirable features. These features include a decade scaler, a time clock, a speaker and an automatic device which stops the scaler and the clock. This combination makes it possible to time a desired number of electrical pulses automatically, to count the number of pulses over a desired time interval, to check by audible means the counting rate and to reduce human error in computation.

The decade scaler is essentially that published by John Potter.[1] It consists of four decades which count electrical impulses and register the count on neon bulbs mounted on the front panel (Fig. 1). This system registers up to 10,000 pulses *but* an electro-mechanical impulse recorder can be plugged in to extend the counting range.

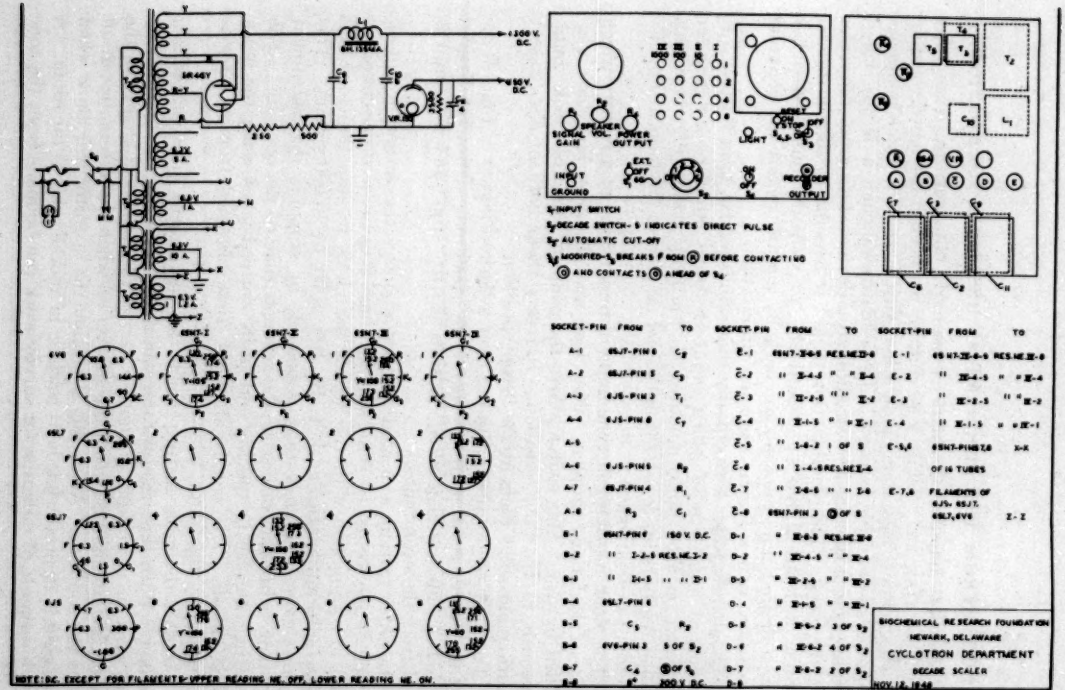
The clock measures in time units of hundredths of a minute which makes calculations simpler. The clock and the neon bulbs are controlled by $S_{4,5}$. This switch starts the counting and the clock simultaneously in the "on" position. It resets them both at the "reset" position and stops them at the "stop" position.

To count the number of pulses over a desired period of time $S_{4,5}$ is put in "reset" position and then in "on" position. After the desired period of time has elapsed $S_{4,5}$ is switched to "stop" and the count is read from the neon bulbs.

In order to time a desired number of pulses the automatic cut-off switch S_3 may be used. This switch, which is in the recorder circuit, is used in conjunction with the decade switch S_2 to stop the clock and neon bulbs automatically at 1, 10, 100, 1000, or 10,000 counts. This is done more accurately than is possible by stopping the instrument manually with $S_{4,5}$ because of the reaction time of the operator. The counting rate can be quickly and easily derived when using the automatic cut-off. The reciprocal of the time obtained from a table is multiplied by the count, which is a power of ten.

Another desirable feature which this instrument incorporates is a speaker. With this, the operator can detect a change in the counting rate while he is working in another part of the room. A change in the circuit is contemplated to cut the speaker off when the automatic cut-off stops the counting. With this alteration the operator will know when the count is stopped while he is working in another part of the room. A rough estimate of radiation for survey work can be made from the tone

[1] *Electronics*, 17:110 (1944).



SOCKET-PIN	FROM	TO
A-1	6SJ7-PIN 6	C_2
A-2	6SJ7-PIN 5	C_3
A-3	6J5-PIN 3	T_1
A-4	6J5-PIN 8	C_7
A-5		
A-6	6J5-PIN 5	R_3
A-7	6SJ7-PIN 4	R_1
A-8	R_3	C_1
B-1	6SN7-PIN 9	150 V. D.C.
B-2	"	2-2-5 RES. MF. 2-2
B-3	"	2-4-5 " " 2-1
B-4	6SL7-PIN 8	
B-5	C_5	R_2
B-6	6V6-PIN 3	5 OF S_2
B-7	C_4	(S) OF S_4
B-8		300 V. D.C.

SOCKET-PIN	FROM	TO
C-1	6SN7-2-6-5 RES. MF. 2-0	
C-2	" 2-4-5 " " 2-4	
C-3	" 2-2-5 " " 2-2	
C-4	" 2-1-5 " " 2-1	
C-5	" 1-6-2 1 OF S	
C-6	" 1-4-5 RES. MF. 1-6	
C-7	" 1-6-5 " " 1-0	
C-8	6SN7-PIN 3 (S) OF S	
D-1	" 2-6-5 RES. MF. 2-0	
D-2	" 2-4-5 " " 2-4	
D-3	" 2-2-5 " " 2-2	
D-4	" 2-1-5 " " 2-1	
D-5	" 2-6-2 3 OF S_2	
D-6	" 2-6-2 4 OF S_2	
D-7	" 2-6-2 2 OF S_2	
D-8		

SOCKET-PIN	FROM	TO
E-1	6SN7-2-6-5 RES. MF. 2-0	
E-2	" 2-4-5 " " 2-4	
E-3	" 2-2-5 " " 2-2	
E-4	" 2-1-5 " " 2-1	
E-5,6	6SN7-PINS 2,8 OF 16 TUBES	X-X
E-7,8	FILAMENTS OF 6J5- 6SJ7, 6SL7, 6V6	I-I

BIOCHEMICAL RESEARCH FOUNDATION
NEWARK, DELAWARE
CYCLOTRON DEPARTMENT
DECADE SCALER
NOV. 12, 1948

FIG. 1. Power supply and service notes.

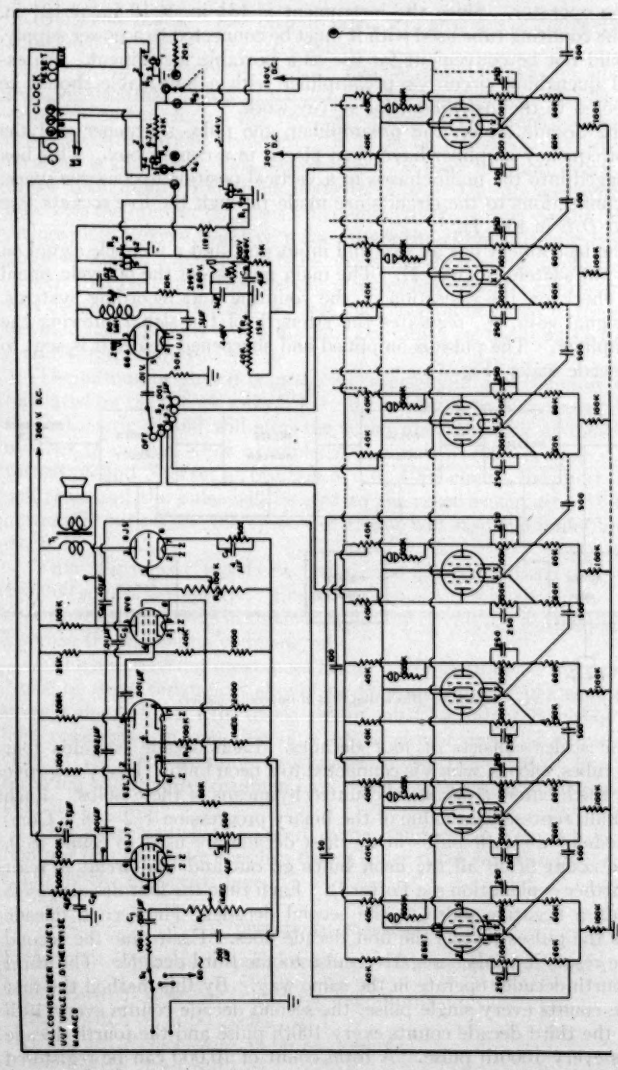


FIG. 2. Schematic diagram of impulse counter.

by the operator. Since the instrument is 13¼ in. × 19 in. × 12¼ in. and the counting tube used with it must be connected to a power supply, it would not be convenient for use as a portable instrument. An external quenching circuit or preamplifier with a long cable should be connected to the instrument for survey work.

The decade scaler, the preamplifier, the pulse sharpener and the audiofrequency amplifier have been placed in a copper box. This box is plugged into the main chassis in a vertical position to conserve space. The connections to the circuits are made through the five sockets (see *A B* $\bar{C}$ *D E* in Fig. 1).

The instrument has an external input post and a 60 cycle signal on the input switch, S_1 (Fig. 2). The main purpose of the 60 cycle signal is to check on the operation of the counting and recording systems. The signal gain, R_1, regulates the strength of the signal entering the preamplifier. The pulse is amplified and sharpened before it is sent to the decade scaler (Fig. 3).

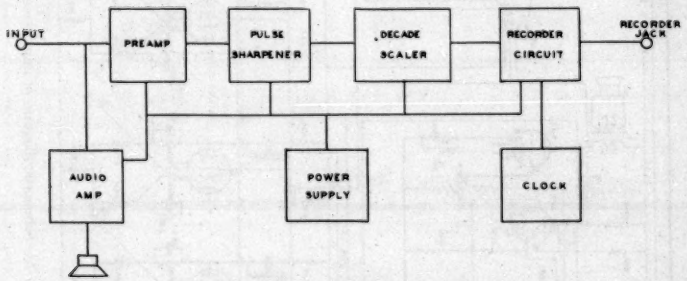


FIG. 3. Block diagram of impulse counter.

The scaler consists of four decades. Each decade contains four 6*SN*7 tubes, each of which is connected to a neon bulb. Every negative pulse which enters a decade is counted by means of these bulbs. Each neon bulb represents a value of the binary progression 1–2–4–8. Combinations of the four bulbs in the first decade are used to count to 9. At the count of 10 all the neon bulbs go out and the circuit is reset (for further explanation see Potter [1]). Each time the first decade resets it sends a negative pulse to the second decade. The second decade counts the pulses just as the first decade does. Each time the second decade resets it sends a negative pulse to the third decade. The third and fourth decades operate in the same way. By this method the first decade counts every single pulse, the second decade counts every 10th pulse, the third decade counts every 100th pulse and the fourth decade counts every 1000th pulse. A total count of 10,000 can be registered

on the neon bulbs. The output post, P_1, provides a connection for additional decades if they are desired.

An electro-mechanical recorder can be inserted into an insulated jack on the front panel to extend the counting range (Fig. 1). The cathode resistor, R_4, of the recorder circuit is necessarily variable to accommodate various recorders. It is to be adjusted only when the recorder is changed. This recorder may be used to count every pulse, every 10th, 100th, 1000th or 10,000th pulse. The decade switch, S_2, regulates the number of pulses which enter the recording circuit (Fig. 2). At position 5 every negative pulse enters the circuit directly from the pulse sharpener circuit and is recorded by the electro-mechanical recorder. At position 1 the recorder receives one pulse on the reset of the first decade; at position 2 it receives one pulse on the reset of the second decade; and positions 3 and 4 admit one pulse on the reset of decades 3 and 4, respectively.

The automatic cut-off is in the recorder circuit and therefore is also regulated by the decade switch, S_2. By setting S_2 to the proper position the automatic cut-off will stop the scaler and the clock at the desired number of counts. For example, if a count of 1000 is desired, S_3 is turned on and S_2 is set at position 3. At 1000 counts the third decade will reset and the pulse will be sent to the recorder circuit. With the automatic cut-off on, the system of relays will stop the scaler and the clock.

This instrument has been tested and proved satisfactory up to 100,000 counts per minute. For most purposes this is sufficient. However further alterations in the preamplifying circuit are contemplated to increase the maximum counting rate.

Unfortunately the resistances in the 6*SN*7 circuits are very critical. None of these resistances should be off more than 5 per cent of the given value. Each twin triode circuit must be balanced within itself. The resistances in one side of a 6*SN*7 circuit must be within 2 per cent of the corresponding resistances in the other side. For easier servicing the voltage readings have been taken on most of the tubes (Fig. 1).

This instrument is to be used at the Biochemical Research Foundation not only to count impulses from Geiger tubes but also to check the r.p.m. of the ultracentrifuge.

OPERATIONAL PROCEDURE.

I. Without electro-mechanical recorder, without automatic cut-off.

(*A*) Switch S_6 to "on", wait for one minute.
(*B*) Switch S_1 to 60 cycles.
(*C*) Turn R_1 clockwise as far as possible.
(*D*) Switch S_2 to "off".

(*E*) Check operation by switching $S_{4,5}$ to "reset", then to "on". After one minute switch $S_{4,5}$ to "stop". The count should be 3600. Switch $S_{4,5}$ to "on".
(*F*) Connect lead from the desired impulse source to the input post.
(*G*) Switch S_1 to external input.
(*H*) Turn R_1 counterclockwise as far as possible. Slowly turn R_1 clockwise until the lights start working. Then turn R_1 clockwise an additional 30 degrees so that the lights will work correctly.
(*I*) Adjust speaker volume R_2.
(*J*) Switch $S_{4,5}$ to "reset" then to "on". At the end of the desired time interval or the desired number of counts switch $S_{4,5}$ to "stop".
(*K*) To make additional counts repeat (*J*).

II. With electro-mechanical recorder, without automatic cut-off.

(*A*) Follow I (*A*, *B*, *C*).
(*B*) Switch S_2 to position 1.
(*C*) Switch S_3 to "off".
(*D*) Plug in recorder.
(*E*) Switch $S_{4,5}$ to "on".
(*F*) Turn R_4 counterclockwise as far as possible. Slowly turn R_4 clockwise until the recorder starts to register. Note this point. Continue to turn R_4 clockwise until it reaches a point where the recorder suddenly increases its recording rate. This is due to oscillation of the thyratron (884). Turn R_4 counterclockwise to a position about halfway between these two points. There it should record a pulse at every reset of the first decade.
(*G*) To make a check, switch S_2 to position 4. The recorder should register a pulse at every reset of the fourth decade.
(*H*) Follow I (*F*–*K*).

III. Without electro-mechanical recorder, with automatic cut-off.

(*A*) Follow I (*A*, *B*, *C*) and II (*B*).
(*B*) Switch S_3 to "on".
(*C*) Switch $S_{4,5}$ to reset and then to "on".
(*D*) If the automatic cut-off does not stop the clock and the neon bulbs immediately, turn R_4 in a clockwise direction until it does stop them.
(*E*) To check: switch S_2 to position 2, repeat III (*C*). The clock and the neon bulbs should stop in from 0.02 to 0.03 minutes. Switch S_2 to position 3, repeat III (*C*). The clock and the neon bulbs should stop in from 0.27 to 0.28 minutes.

Switch S_2 to position 4, repeat III (*C*). The clock and the neon bulbs should stop in 2.77 minutes.

(*F*) Turn S_3 to "off".
(*G*) Follow I (*F–I*).
(*H*) Switch S_3 to "on".
(*I*) Switch S_2 to the desired position.
(*J*) Repeat III (*C*).
(*K*) To make another count, switch $S_{4,5}$ to "reset" then to "on".

CURRENT TOPICS.

Fruit and Vegetables Unaffected by Normal Flight Condition.—That the quality of fresh fruits and vegetables moved by air transport is unaffected by sudden changes in air pressure is shown in results of a joint study reported by plant scientists of the U. S. Department of Agriculture and engineers of the Lockheed Aircraft Corporation.

Extensive tests on 34 different fruits and vegetables indicate they are not harmed by normal flight conditions provided the temperature and humidity are controlled.

The tests were conducted in Lockheed's giant altitude chamber at Burbank, Calif. Trays of farm-fresh produce were placed in the interior of a decompression chamber used to simulate flights. This permitted the control of altitude, rate of climb and descent, temperature and humidity.

Tests were made in which simulated climbs were made at the rate of 3000 ft. per minute to an altitude of 30,000 ft. with descents at the same rate. The fruit was then held in a control room at normal marketing conditions for several days to permit comparison with other produce purchased at the same time and held under controlled conditions of temperature and humidity.

In an experiment in which a warm, dry flight was simulated, the produce, which had not been precooled, became dull and wilted after 2 hr. in the test chamber. Loss in weight was high, ranging from 6 to 15 per cent in asparagus, broccoli, green onions, red raspberries, and strawberries.

In another test, fruit and vegetables remained fresh in appearance and lost very little weight when precooled to about 40°F. and humidity held at 55 to 75 per cent.

With only one exception, none of the commodities showed injury that could be attributed to altitude. Simulated rapid ascent at the rate of 5000 ft. per minute to an altitude of 50,000 ft. resulted in the bursting of one tomato and one persimmon out of approximately a dozen of each. No other items were affected.

On the basis of these tests it is believed that pressurization equipment is unnecessary in the transportation of fruits and vegetables by air but that the control of humidity and temperature is important.

In addition to the fruits and vegetables mentioned, tests were made with: avocados, beans (string), beans (lima), beets, bell peppers, brussel sprouts, carrots, cauliflower, celery, cucumbers, egg-plants, figs, grapes, limes, mushrooms, mustard greens, parsley, peaches, peas, pears, radishes, Romaine lettuce, salad mix, spinach, sweet corn, squash, water cress.

The project was undertaken at the request of the U. S. Department of Commerce. The tests were conducted by W. T. Pentzer, W. R. Barger, and E. P. Atrops of the Bureau of Plant Industry, Soils, and Agricultural Engineering, and by B. A. Rose and L. R. Hackney of Lockheed. Costs of the experiments were shared by the Lockheed company and the U. S. Department of Agriculture.

Post-War X-Ray Progress Exceeds That of Previous 20 Years, Executive Reports.—Research scientists, seeking new and better methods of using X-rays, have made as much progress in the three years since the end of World War II as they did in all of the previous 20 years, according to Walter Evans, vice president of the Westinghouse Electric Corporation and head of the company's Electronics and X-Ray Division.

Mr. Evans pointed specifically to the recent completion of basic development on two new X-ray devices. One is an electronic "telescope" to increase by 500 times the brightness of an image on the X-ray fluoroscopic screen; the other is equipment to make X-ray movies of fast-moving objects with exposures 2000 times faster than the human eye can blink.

"Each of these developments represents a giant stride forward in the science of X-rays," said Mr. Evans. "Together they give promise of becoming the greatest achievements in X-ray research in the past 20 years."

Mr. Evans' comments were prompted by a Westinghouse "Stockholders' Quarterly," prepared for the company's stockholders. The Quarterly described the development of X-rays since their discovery by Wilhelm Conrad Roentgen 53 years ago.

The Quarterly announced, "Recently from the Westinghouse Research Laboratories at East Pittsburgh, Pa., came word of a new electronic tube that brings for the first time a significant increase in the brightness of the image seen on the screen of an X-ray fluoroscope. The fluoroscope is a machine that gives physicians a direct X-ray view of their patients without need of photography.

"Result of six years of research, the new tube is called the X-ray 'image amplifier.' It promises to give physicians a 500 times brighter view of a patient's internal organs in action. It has been described as the greatest single advance in X-ray diagnosis since the discovery of these 'telling rays.'"

In operation, the pamphlet explained, X-rays, which have passed through a patient, strike a fluorescent screen, releasing light rays. These light rays illuminate a photo-sensitive surface on the other side of the screen, freeing electrons—tiny particles of negative electricity. The electrons then flash through the tube and strike a second fluorescent screen, releasing a flood of light rays and forming the image viewed by the physician.

Basic development of the image amplifier has been completed and a full sized commercial tube now is being built.

The Quarterly added, "Another development, which like the 'telescope' is just emerging from the research stage, is an X-ray motion picture apparatus that makes exposures of only 10 millionths of a second on film in a shutterless movie camera. This super-speed device will, for the first time, make possible X-ray movies of speeding objects without blur. In industry, it can be used to analyze the internal structure of rapidly moving parts or even flowing metal."

Some other outstanding recent developments mentioned in the Quarterly included:

1. The Westinghouse Micronex, an ultra high-speed unit that makes single X-ray photographs at the fantastic speed of one-millionth of a second. During the war the Micronex was used extensively in the atomic bomb project but its exact purpose was kept secret. It did, however, make possible studies of such

things as the action of a bullet zipping down a gun barrel, or the swelling, bursting, and disintegration of a high explosive shell chewing its way through steel plate.

2. An X-ray thickness gauge for measuring sheet steel flying past at 4000 ft. a minute without making contact with the metal. The gauge is so sensitive it can indicate a change as small as 1/720th of the thickness of a piece of newspaper.

3. Modern photofluorographic units, known as "PFX," that make high-quality, small-film X-ray photographs at about one-sixtieth the cost and in one-fourth the time required by earlier methods. This equipment is used for mass chest surveys conducted by public health and allied groups in the widespread campaigns to control tuberculosis through early diagnosis. Westinghouse manufactures more than half of the photofluorographic units in use today.

The Quarterly concluded, "Output of all types of Westinghouse X-ray equipment in 1947 tripled the best pre-war years and exceeded even the previous war-time peak established in 1944. This high rate of operations has continued in 1948."

USDA and AEC Expand Research with Radioactive Chemicals on Soil and Plant Problems.—Extensive new programs of research in soil and plant science will be carried on under arrangements concluded by the U. S. Department of Agriculture and the U. S. Atomic Energy Commission. The programs will center in new facilities, such as greenhouses and "hot" laboratories for handling radioactive chemicals, to be constructed by the Atomic Energy Commission at the Plant Industry Station of the Department at Beltsville, Md. Soil and plant scientists on the staff of the Bureau of Plant Industry, Soils, and Agricultural Engineering will carry on the studies.

One group of studies is aimed to give aid in the safe disposal of liquid radioactive wastes at installations of the atomic energy program by ascertaining exactly how radioactive elements behave when introduced into soils. Facts will be sought on the movement, the fixation, and the release of various radioactive substances in various soil types.

These studies are expected to result in new and more precise knowledge which will bear on how and when to plow and cultivate and fertilize for best results with different crops on different soil types.

Other purposes of the work to be carried on under the agreement include:

1. Development of procedures for safe and effective use of radioactive isotopes in soil and crop research.

2. Supplying for use by other agricultural research agencies fertilizers incorporating radioactive trace elements.

3. Developing in the U. S. Department of Agriculture and co-operating State agricultural experiment stations a group of scientists skilled in the use of isotopes in soil and crop research.

In the fundamental soil research the scientists will study the mechanism by which nutrients in soil are released and transferred to plants. They will make similar studies of fertilizer and liming materials added to the soil. Much

of this work will be with the so-called secondary and minor or trace elements—cobalt, molybdenum, copper, manganese, etc. Biochemical investigations are intended to throw light on the part that various chemical elements play in growth of plants, how plants take up nutrients from the soil, the relation between the chemical composition of plants and the soil in which they grow, or of fertilizers used on the soil.

The Bureau of Plant Industry, Soils, and Agricultural Engineering will provide an area at the station on which the special facilities, to cost about $200,000, will be constructed. The facilities will include a greenhouse and a headhouse with a full basement containing constant-environment growth rooms. The headhouse will have so-called "hot" laboratories designed for handling radioactive isotopes. There will also be improved facilities for the manufacture of radioactive fertilizers, soil amendments such as lime and related materials. Adjacent to the greenhouse will be lysimeters for use in the study of the movement of radioactive materials in the soils. Soil frames will be available for small-scale field experiments.

The improved facilities for the manufacture of radioactive fertilizers and soil amendments will be used to produce these materials for State agricultural experiment stations and other research agencies. The Bureau has done this on a limited scale during the past year but can increase production when the new facilities are completed.

It is expected that arrangements will be made with the Graduate School of the Department of Agriculture whereby graduate students may receive training at the Plant Industry Station in the use of radioactive isotopes. Likewise, arrangements will be made for staff members of other agricultural research agencies to obtain experience on the project at Beltsville.

Uranium Supplies. (*Excerpts from Statement of David E. Lilienthal, Chairman, U. S. Atomic Energy Commission*).—The potentialities of the atomic age have been sold short several times in recent weeks. Statements have been made and widely publicized that there is only enough uranium ore to last a relatively brief period, and so the prospects of benefit from atomic power must go glimmering. This is simply not so. It appears clear that atomic energy is on a sound basis for an indefinite period in the future.

An unwarranted impression that a shortage of uranium will drastically limit the possibilities of atomic energy has been caused by estimates that known reserves of uranium would last perhaps only 30 years or not more than 40. Estimates of the total world supply of uranium have run from 30,000 tons to 500,000 tons, seventeen times as much. Estimates of rates of potential consumption of the fissionable material derived from uranium have also varied considerably.

The people of the United States are entitled to know that these estimates of a short-lived atomic enterprise are not correct. If they were true, this would be very serious news. As explicitly as national security permits, the Atomic Energy Commission wishes to state that it finds no basis in fact for these statements about extremely limited uranium ore supplies. If this foreshortening of the atomic age were accurate, the plants and operations of the atomic energy program of the United States would have to close up in a few years.

Instead, the contrary is true about uranium supplies. Estimated known reserves have already increased substantially since usefulness of uranium as a source of fissionable material became firmly established.

Let us look at the facts as they are available generally for making such estimates. Necessarily most of the published data concerning ore deposits and ore reserves is pre-war information. Before the war, uranium was used principally as a pigment in the ceramics industry and was essentially a by-product of radium or vanadium ore. Man has just begun to look for uranium; in every quarter of the earth's surface prospectors' picks are breaking the rocks in this search. The current great prospecting surge may result in the discovery of new uranium deposits that will significantly alter our concept of world supply. Some good prospects are turning up in Canada, to cite only one country. It would be surprising if new discoveries there would not, over the years, equal or better the great Eldorado mine. Only a few new important discoveries would change the world supply picture profoundly.

Nor is it necessary that such new discoveries be made in hitherto unsuspected places. One of the rules of mining is to look for new ore deposits close to known big ones. New discoveries in the Eldorado mine or Shinkolobwe mine in the African Congo would affect the uranium supply position of the world as profoundly as, say, one in South America—and would be reflected in production sooner.

Another rule is that good mines die hard. The comparison of measurable ore reserves with rates of consumption at any time means very little in terms of future supply. The prophets of early exhaustion of our petroleum, lead, copper, and other non-replaceable raw materials have found this out to their chagrin generation after generation.

Sweden has already announced that she plans to recover uranium from oil shales and other types of marine sediments which contain small amounts of uranium are under study in this country as in other parts of the world. One important potential low-grade source is that announced recently by the Union of South Africa. The gold ores of the Witwatersrand contain low concentrations of uranium, and by-product uranium undoubtedly will come from the huge gold mining industry of that country.

Uranium has been estimated to occur in the crust of the earth probably a thousand times as plentifully as gold or a hundred times as plentifully as silver, and possibly more plentifully than lead or zinc. Figures of this kind, however, have even less meaning than figures for proven ore reserves. It takes considerable concentration of uranium, just as it does of lead or zinc, to make a mineable deposit under anything like present economic conditions. The fact, however, that uranium is not a great rarity lends credence to a geological opinion that it is apt to be present in the right place reasonably often when geological concentrating processes are at work.

The Atomic Energy Commission of the United States is responsible for one of the most extensive and intensive searches for a mineral that has ever been conducted, and on a world-wide basis. We do not expect that there will be an unlimited supply but there is no sound basis for a conclusion that usable uranium ore supplies will not be available for the indefinite future.